



Say we are assigned complex numbers A and B represented by $r_A e^{i\alpha}$ and $r_B e^{i\beta}$ respectively, and we know them to be also vertices of a certain polygon ABCD suspended in complex Argand space, how then can we discover points C and D if the quantities r_1 , r_2 , θ_1 and θ_2 are also known?

Solving this through the use of vectors is an elegant approach to say the least.

Let $\vec{OA} = a$, $\vec{OB} = b$, $\vec{OC} = c$ and $\vec{OD} = d$

Firstly, compute $\vec{AB} = \vec{OB} - \vec{OA} = b - a = r_B e^{i\beta} - r_A e^{i\alpha} = R e^{i\phi}$,

where $|b - a| = R$ and $\arg(b - a) = \phi$

Geometrically speaking, if we multiply the vector \vec{AB} by a general complex number say $\ell e^{i\delta}$, then the magnitude of this vector will be **scaled** by ℓ units and **rotated** through δ radians. (clock-wise or anti-clock-wise depending on whether it is designated positive or negative)

So $\vec{AC} = \vec{AB}$ multiplied by $\frac{r_1}{R} e^{i\theta_1} = R e^{i\phi}$ multiplied by $\frac{r_1}{R} e^{i\theta_1}$ and

$\vec{AD} = \vec{AB}$ multiplied by $\frac{r_2}{R} e^{i\theta_2} = R e^{i\phi}$ multiplied by $\frac{r_2}{R} e^{i\theta_2}$

As such, we can now **attempt to solve** for complex numbers C and D by recognizing that

$$\vec{OC} = \vec{OA} + \vec{AC} = a + c \text{ and } \vec{OD} = \vec{OA} + \vec{AD} = a + d$$