



How can we prove that a triangle inscribed within a semi-circle is always a right angled one using a vectors method? Here goes:

Firstly, by assigning the centre of the semi-circle to be the origin, we define the general vectors $\vec{OA} = a$, $\vec{OB} = b$; in addition, recognise that $|a| = |b|$ since both lengths **represent radii of the same semi-circle**.

Based on the above diagram, we have $\vec{AB} = b - a$, $\vec{AC} = -a - b$

Then $\vec{AB} \cdot \vec{AC} = (b - a) \cdot (-a - b) = -a \cdot b - b \cdot b + a \cdot a + a \cdot b$

$$= -b \cdot b + a \cdot a$$

$$= -|b|^2 + |a|^2 = 0 \quad (\because |a| = |b|)$$

This therefore implies angle $\hat{AOB} = \frac{\pi}{2}$ (shown)