

When solving for roots of higher degree polynomials in complex numbers, it is essential to convert $e^{i\theta}$ into its equivalent form $e^{i(2k\pi+\theta)}$ so that all roots can be discovered as the value of the **integer** k changes.

So how is $e^{i(2k\pi+\theta)} = e^{i\theta}$?

I will provide the mathematical proof below, which is actually rather simple:

$$\begin{aligned}
 e^{i(2k\pi+\theta)} &= \cos(2k\pi + \theta) + i \sin(2k\pi + \theta) \\
 &= \cos(2k\pi) \cos \theta - \sin(2k\pi) \sin \theta + i [\sin(2k\pi) \cos \theta + \cos(2k\pi) \sin \theta] \text{----- (1)}
 \end{aligned}$$

For all $k \in \mathbb{Z}$, $\cos(2k\pi) = 1$ and $\sin(2k\pi) = 0$;

Hence (1) reduces to $\cos \theta + i \sin \theta = e^{i\theta}$ (shown)

(Note: the trigonometric expansions $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ were employed in the above workings.)