

Hypothesis Testing Summary

Hypothesis testing is typically employed to establish the authenticity of claims based on referencing specific statistical parameters including the **level of significance**; in this regard there are no absolute “truths” or “lies”.

The null hypothesis is set up as such:

$$H_0 : \mu = \mu_0$$

The alternate hypothesis is formulated depending on whether a one-tail or two-tail test is required:

$$H_1 : \mu > \mu_0 \quad \text{or} \quad H_1 : \mu < \mu_0 \quad \longrightarrow \quad \text{one-tail test}$$

(Note: Keywords such as **underestimate**, **overestimate**, **overstating**, **understating** etc would be featured in the problem.)

$$H_1 : \mu \neq \mu_0 \quad \longrightarrow \quad \text{two-tail test}$$

(Note: Keywords such as **change**, **differ** /**difference** etc would be featured in the problem.)

Level of significance $\alpha\%$: there is a probability of $\frac{\alpha}{100}$ where H_0 is wrongly rejected in

favour of H_1 .

Z-test

Upon clearly identifying the values of \bar{x} , n , σ^2 and μ_0 within the problem, run the Z -test with test statistic $Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ based on $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ (take care to append CLT as the basis of approximation if the underlying distribution of X is non-normal and sample size involved is sensibly large).

If the resulting p -value is smaller than $\frac{\alpha}{100}$, then H_0 is rejected and there is sufficient evidence to suggest that H_1 is correct at the $\alpha\%$ level.

If the resulting p -value is larger than $\frac{\alpha}{100}$, then H_0 is accepted and there is insufficient evidence to suggest that H_1 is correct at the $\alpha\%$ level.

t-test

Three conditions must be concurrently satisfied in order for this version of the test to be used:

- (i) The underlying distribution is normal.
- (ii) The population variance is unknown. (Hence an unbiased estimate s^2 must be calculated.)
- (iii) The sample size under investigation is small. ($n \leq 15$ would be an acceptable range.)

If conditions (ii) and (iii) are explicitly indicated within the problem, yet no mention is made of whether the original population is normally distributed, then it can be assumed as such (and the t -test subsequently carried out) if the problem includes a supplementary clause calling for suitable assumptions to be made.

Upon clearly identifying the values of \bar{x} , n , s^2 and μ_0 within the problem, run the t -test with test

statistic $T = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ and $\nu = n - 1$, where ν denotes the degrees of freedom.

If the resulting p -value is smaller than $\frac{\alpha}{100}$, then H_0 is rejected and there is sufficient evidence to suggest that H_1 is correct at the $\alpha\%$ level.

If the resulting p -value is larger than $\frac{\alpha}{100}$, then H_0 is accepted and there is insufficient evidence to suggest that H_1 is correct at the $\alpha\%$ level.

Special Mentions

In the event a problem DOES NOT state the level of significance to be referenced against in the testing process, but instead demands the minimum level of significance (in percent) such that H_0 is rejected, it can be achieved by simply pegging this to the p -value, ie $\alpha_{\min} = 100p$.

In the event a problem requires the student to **discover the range of values** for one of the four quantities (\bar{x} , n , σ^2 and μ_0) based on a pre-established conclusion, then consider the following sequence of steps:

1. Draw the standard normal distribution curve $Z \sim N(0, 1)$.
2. Pencil in the rejection regions based on whether a one-tail or two-tail test is required.

3. For $\alpha\%$ level of significance, find Z_{crit} such that

$$P(Z_{crit} < a) = \frac{\alpha}{100} \quad \text{for a left-tail test;}$$

$$P(Z_{crit} > a) = \frac{\alpha}{100} \quad \text{for a right-tail test;}$$

$$P(Z_{crit} < a) = \frac{1}{2} \left(\frac{\alpha}{100} \right) \quad \text{for a two-tail test.}$$

4. Write out the test statistic $Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ and pit this against Z_{crit} to obtain the final necessary inequality.

For example, if a left-tail Z-test is used, and the student is tasked to find the range of values of \bar{x} such that H_0 is rejected (ie the test –statistic MUST therefore reside in the rejection region),

$$\text{then we simply set } \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} < Z_{crit} \Rightarrow \bar{x} < (Z_{crit}) \frac{\sigma}{\sqrt{n}} + \mu_0.$$

Note: The above illustration is constructed assuming a Z -test is used; if a t -test is instead required, the working steps are exactly identical except the t distribution curve and the test statistic

$$T = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \text{ are now considered.}$$

FULLY WORKED EXAMPLE:

QNS:

A random sample of 100 wooden rods of a certain length is measured and the length x in metres is recorded. The results are summarised as follows:

$$\sum(x - 30) = -150, \quad \sum(x - \bar{x})^2 = 1485.$$

A test was carried out at the 2% significance level with the following hypotheses:

H_0 : The population mean length of the wooden rod is μ_0 .

H_1 : The population mean length of the wooden rod is not equal to μ_0 .

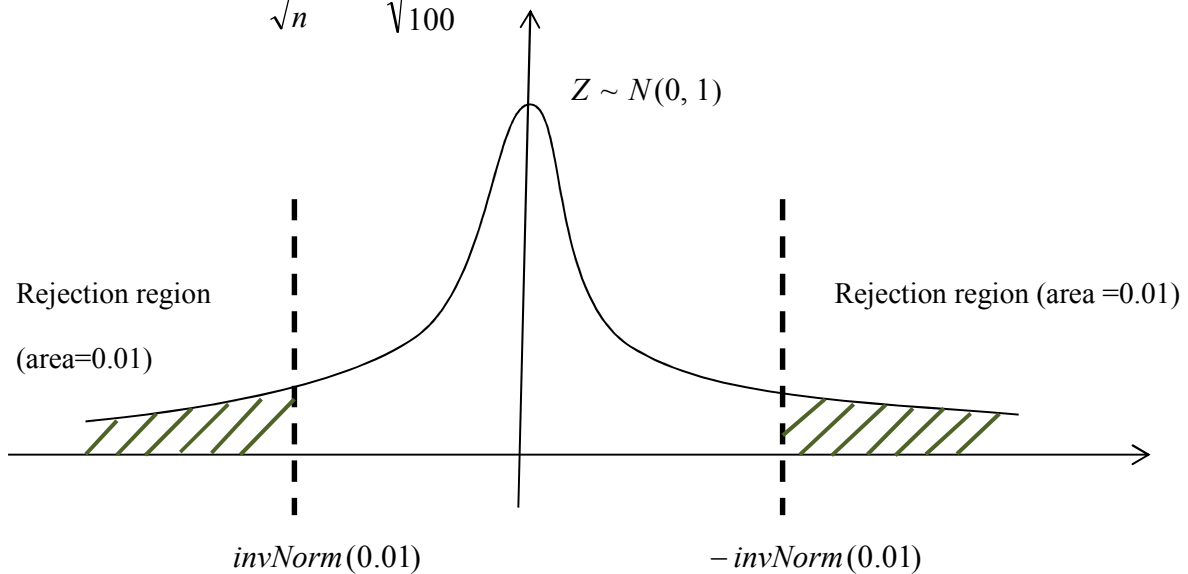
Given that H_0 is rejected in favour of H_1 , find the set of possible values of μ_0 . Assume that the distribution of the length of wooden rods is normal.

SOLUTION :

Unbiased estimate of population mean $= \bar{x} = -\frac{150}{100} + 30 = 28.5$

Unbiased estimate of population variance $= s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2 = \frac{1485}{99} = 15$

Test statistic $Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{28.5 - \mu_0}{\sqrt{\frac{15}{100}}}$



Since H_0 is rejected in favour of H_1 ,

$$\frac{28.5 - \mu_0}{\sqrt{\frac{15}{100}}} < \text{invNorm}(0.01) = -2.326 \quad \text{or} \quad \frac{28.5 - \mu_0}{\sqrt{\frac{15}{100}}} > -\text{invNorm}(0.01) = 2.326$$

ie $\mu_0 > 29.4$ or $\mu_0 < 27.6$ (shown)