Functions Summary

1. Definitions

Domain (D_f) : set of x values for which the function $f(x)$ is defined.

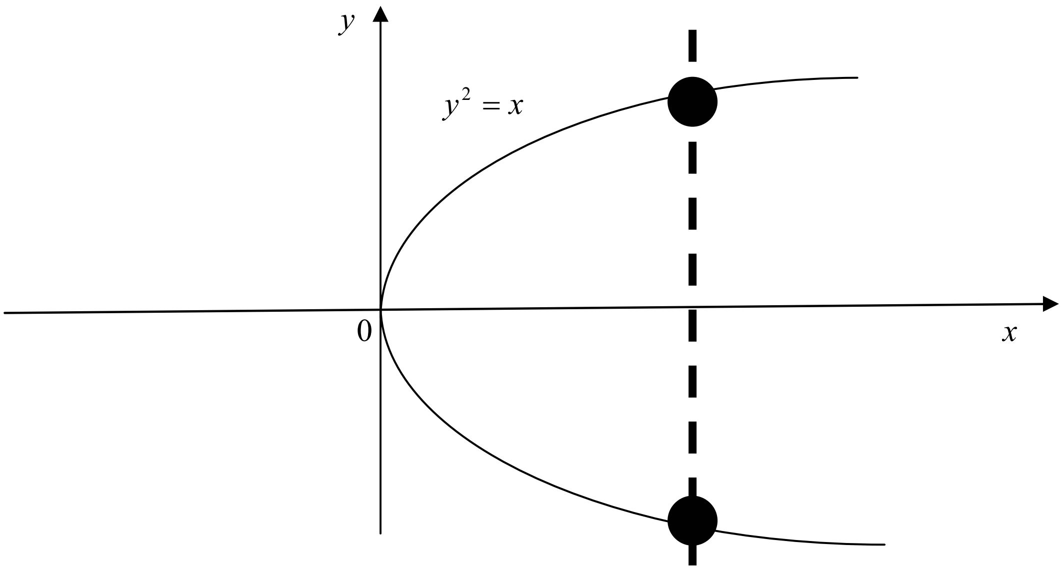
Range (R_f) : set of y values for which the function $f(x)$ is defined.

For completeness of description, any function definition must include a **reference domain**, eg $f : x \to x^2 + 2$, $x \geq 0$

2. Vertical line test

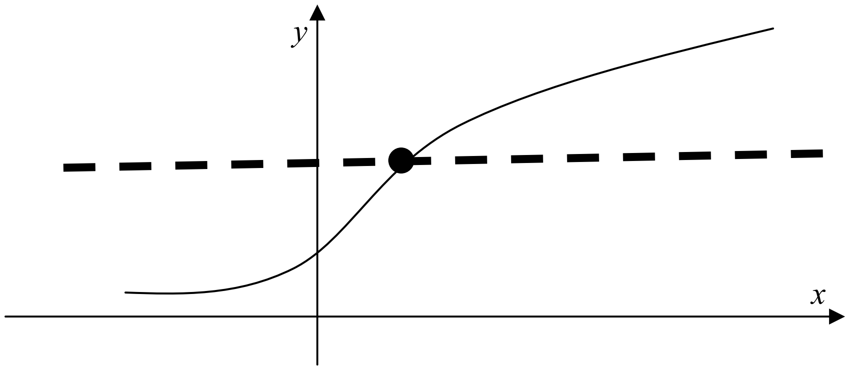
A function is properly defined if no two unique distinct values of y are associated with the same x value. A vertical line test aids in ascertaining this-if any vertical line drawn **cuts through the graphical representation** of the candidate definition at **two or more points**, then that definition is **not a function**.

Example: $y^2 = x$ is not a function because a vertical line drawn typically cuts through its graph at two points.(see below)



3. Horizontal line test

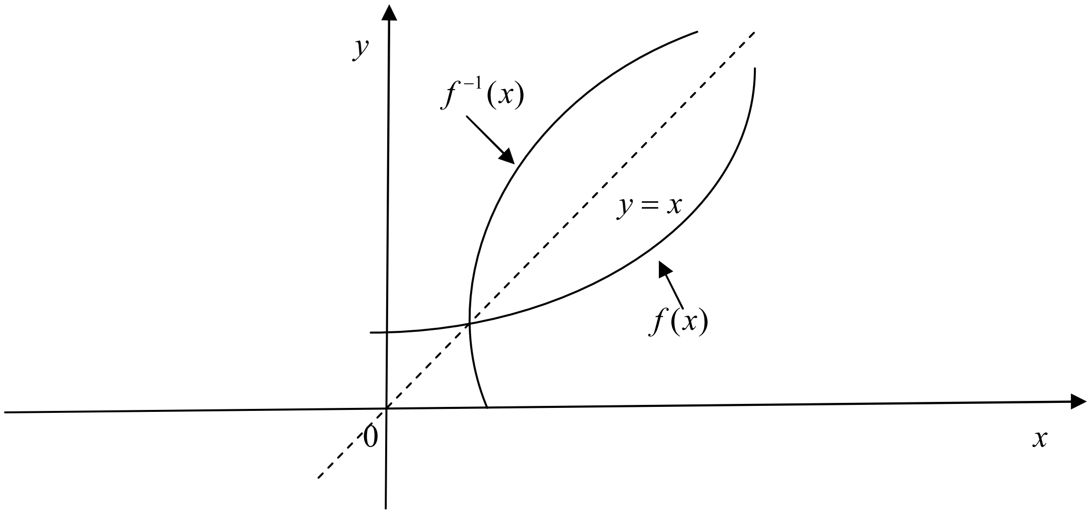
A function is said to be **injective** (1-1) if every element within its domain maps directly to a unique, distinct image, ie every x value yields a unique corresponding y value. This can be verified via the horizontal line test, ie a horizontal line drawn at any level will **only cut the function once**(see below).



Note: functions that are **strictly increasing/decreasing** in the specified domain will always be 1-1.

4. Inverse functions

A function $f(x)$ is said to have an inverse $f^{-1}(x)$ if it is 1-1. This inverse function is obtained graphically by **reflecting** the original function in the line $y = x$.



Properties: $R_{f^{-1}} = D_f$, $D_{f^{-1}} = R_f$

To find an expression for $f^{-1}(x)$:

(i) Let $y = f(x)$

(ii) Through manipulation, make x the subject such that $x = g(y)$

(iii) Replace the function in y entirely with x, ie $f^{-1}(x) = g(x)$

Note: (a) If $f(x)$ is of the form $ax^2 + bx + c$, completing the square is required to obtain x in terms of y.

(b) If $f(x) = f^{-1}(x)$ is required to be solved, it is typically more convenient to simply solve for x through the formulation of $f(x) = x$.

(c) Functions that originally do not possess an inverse can have their domains shrunk such that an inverse function becomes realisable.
Example: $y = x^2$, $x \in \Re$ does not have an inverse, but
$y = x^2$, $x \geq 0$ has an inverse.

5. Composite functions

A composite function of the form $fg(x)$ is said to exist if the range of $g(x)$ is a subset of the domain of $f(x)$, ie $R_g \subseteq D_f$. Note that the domain of $fg(x)$ is simply the domain of $g(x)$, ie $D_{fg} = D_g$.

To determine the range of the composite function $fg(x)$:

Method 1

(i) Find $R_g \cap D_f$

(ii) The result from (i) will serve as the input domain to $f(x)$.

(iii) Obtain the corresponding range of $f(x)$ based on the new domain in (ii). This range is there fore the range of $fg(x)$.

Method 2

(i) Recognise that $D_{fg} = D_g$.

(ii) Find an expression for $fg(x)$, and subsequently its graphical representation.

(iii)Use D_g as the input domain for $fg(x)$, and subsequently obtain the corresponding range graphically. This range is there fore the range of $fg(x)$.