If one is presented with both the definition of $f(x)$ and that of the composite function $fg(x)$, how exactly can the unknown function $g(x)$ be discovered? This is assuming at the onset that the criterion for the existence of $fg(x)$ is satisfied, ie $R_g \subseteq D_f$. The provision of an actual example best illustrates this.

Eg Let $f(x) = 2x - 5$ and $fg(x) = x^2 - 7x + 11$, then

we can say that $fg(x) = 2g(x) - 5 = x^2 - 7x + 11$

Doing a little shuffling of terms therefore gives $g(x) = \frac{1}{2}\left[\left(x^2 - 7x + 11\right) + 5\right]$

$$= \frac{1}{2}x^2 - \frac{7}{2}x + 8 \text{ (shown)}$$

Unsure if the answer arrived at is accurate? We can always work backwards:

$fg(x) = f\left(\frac{1}{2}x^2 - \frac{7}{2}x + 8\right) = 2\left(\frac{1}{2}x^2 - \frac{7}{2}x + 8\right) - 5 = x^2 - 7x + 11$, so things are correct.

Say, what if a **reverse of sorts** was offered in the question, ie to find the unknown function $g(x)$ if instead $f(x)$ and the composite function $gf(x)$ are given? The solving process isn't all that hard either. Let's consider the above example once again, with a slight modification:

Eg Let $f(x) = 2x - 5$ and $gf(x) = x^2 - 7x + 11$, then

we can say $gf(x) = g(2x - 5) = x^2 - 7x + 11$

In this case, we replace x by $\frac{1}{2}x + \frac{5}{2}$ such that $2x - 5 = 2\left(\frac{1}{2}x + \frac{5}{2}\right) - 5 = x$

Thus, $g(x) = \left(\frac{1}{2}x + \frac{5}{2}\right)^2 - 7\left(\frac{1}{2}x + \frac{5}{2}\right) + 11$

$$= \frac{1}{4}x^2 - x - \frac{1}{4} \text{ (shown)}$$

A quick check to verify the above solution is correct:

$gf(x) = \frac{1}{4}(2x - 5)^2 - (2x - 5) - \frac{1}{4} = \frac{1}{4}\left(4x^2 - 20x + 25\right) - 2x + 5 - \frac{1}{4} = x^2 - 7x + 11$.