1 The plane Π has equation $2x-3y+6z=7$ and the line l has equation $\mathbf{r}=8\mathbf{i}-2\mathbf{j}+8\mathbf{k}+t(3\mathbf{i}-\mathbf{j}+2\mathbf{k})$.

The points A and B on l are given by $t=4$ and $t=2$ respectively.

(i) Find the angle ABO.

(ii) Find the coordinates of the foot of perpendicular from A to Π.

(iii) Find the acute angle between Π and the plane $x=0$.

(iv) The point on l where $t=\lambda$ is denoted by P. Find the set of values of λ for which the perpendicular distance from P to Π is less than 3.

2 The position vectors of A and B with respect to the origin O are **a** and **b** respectively. The point C is such that $OACB$ is a parallelogram. The point P on AC is such that $AP : PC = 1 : 2$ and the point Q on BC is such that $BQ : QC = 1 : 2$.

(i) Find $\overrightarrow{QP}$ in terms of **a** and **b**.

(ii) Show that $\overrightarrow{OC}\cdot\overrightarrow{QP}$ can be written as $\lambda|\mathbf{a}|^2+\mu|\mathbf{b}|^2$, where λ and μ are constants to be found.

(iii) Given that $\overrightarrow{OC}\cdot\overrightarrow{QP}=0$, identify the shape of the parallelogram $OACB$, justifying your answer.

3 The equations of three planes p_1, p_2 and p_3 are

$$2y-z=0,$$
$$\beta x+z=2,$$
$$x+\lambda y-2z=\mu$$

respectively, where β, λ and μ are constants.

Relative to the origin O, the points A and B have position vectors given by $4\mathbf{k}$ and $3\mathbf{j}$ respectively.

(i) Find the acute angle between p_1 and the z-axis. Hence or otherwise, find the exact distance from the point A to p_1.

(ii) A plane p_4 is parallel to the plane p_1 such that the distance of p_4 from the point B is twice that of the distance of p_1 from the point B. Find the two possible vector equations of p_4, in scalar product form.

(iii) Verify that the point with coordinates (0, 1, 2) lies on the planes p_1 and p_2. The planes p_1 and p_2 intersect in a line l. Find the equation of the line l in terms of β.

(iv) Given that $\beta=2$ and the three planes p_1, p_2 and p_3 have no point in common, what can be said about the values of λ and μ?