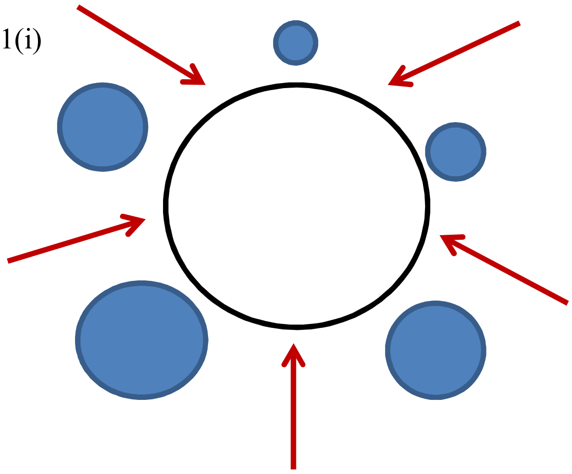
Additional Permutation And Combination Questions Solutions

1(i)



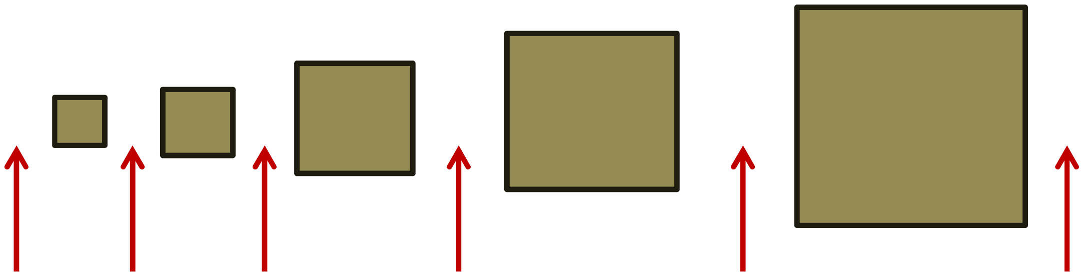
Number of ways to arrange the circles in a circular fashion $(5-1)! = 24$

Since all 5 squares must be inserted in accordance to the 5 arrows denoted above,

Number of ways to arrange the squares $= 5! = 120$

Hence, total number of possible arrangements $= 120 \times 24 = 2880$ (shown)

(ii)



Firstly, number of ways to arrange the 5 squares $= 5! = 120$

There are 6 possible slots to insert the 3 triangles (as denoted by the red arrows)

Number of ways to arrange the triangles $= {}^{6}C_{3} \times 3! = 120$

Hence, total number of possible arrangements $= 120 \times 120 = 14400$ (shown)

2(i) Total number of ways $= {}^{30}C_{3} \times 3! = 24360$ (shown)

(ii) Number of ways such that there one contestant wins both titles = 30

Number of ways such that two titles are won by separate contestants = $30 \times 29 = 870$
(or one could use $= {}^{30}C_{2} \times 2! = 870$)

Total number of ways =870+30= 900 (shown)

3(i) Let the people be labelled A_1, A_2, A_3,................ A_{39}, A_{40}

Starting with A_1, he would have made 39 unique handshakes with others.

Moving on to A_2, he would have made 38 unique handshakes with others.

(discounting the duplicate case of him having shook hands with A_1)

Moving on to A_3, he would have made 37 unique handshakes with others.

(discounting the duplicate cases of him having shook hands with A_1 and A_2)

This process continues till it reaches A_{40}, by then he would have made 0 unique handshakes.

Hence, total number of handshakes made $= 39 + 38 + 37 + \ldots\ldots\ldots\ldots + 1 + 0 = \frac{40}{2}(39 + 0)$

$= 780$ (shown)

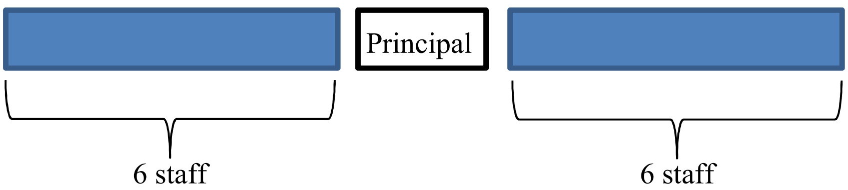
(ii) Grouping each couple as a single unit, we have 5 units altogether.

Number of ways to arrange these 5 units $= 5! = 120$

However, since the husband and wife within each unit can swap places,

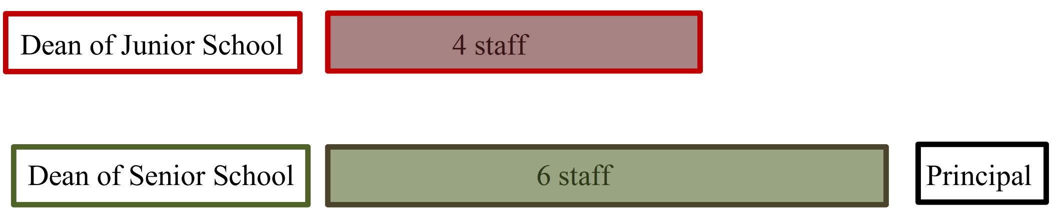
hence total number of possible arrangements $= 120 \times (2!)^5 = 3840$ (shown)

4(i)



Number of ways $= {}^{12}C_6 \times 6! \times {}^{6}C_6 \times 6! = 479001600$ (shown)

(ii) Consider the instance when all staff from the Senior School stand in the first row, while those from the Junior School stand in the second row.



Number of possible arrangements $= 4! \times 6! = 17280$

(Fixing both Deans and Principal in their respective positions and only arranging the remaining staff in the separate rows)

Since the two rows can swap over with the Principal remaining at his rightmost position in the first row, total number of ways $= 17280 \times 2 = 34560$ (shown)

(iii) Total number of ways $= 4! \times 6! \times 2! = 34560$ (shown)

5(i) Since the seats are **numbered**, arrangement around the 3 separate tables would be considered in terms of arranging each set of 4 students in a straight line.

Number of ways $= \underbrace{{}^{12}C_4 \times 4!}_{A} \times \underbrace{{}^{8}C_4 \times 4!}_{B} \times \underbrace{{}^{4}C_4 \times 4!}_{C} = 479001600$ (shown)

(ii) Number of ways $= \underbrace{\left({}^{6}C_2 \times {}^{6}C_2 \times 4!\right)}_{A} \times \underbrace{\left({}^{4}C_2 \times {}^{4}C_2 \times 4!\right)}_{B} \times \underbrace{4!}_{C} = 111974400$ (shown)

(iii) Number of ways $= \underbrace{\left({}^{6}C_2 \times {}^{6}C_2 \times 2^3\right)}_{A} \times \underbrace{\left({}^{4}C_2 \times {}^{4}C_2 \times 2^3\right)}_{B} \times \underbrace{2^3}_{C} = 4147200$ (shown)

6. There are **4** As, **3** Ps, **1** R and **2** Bs.

Case 1: All 3 letters of the code word are the same.

Number of possible ways =2 (AAA and PPP)

Case 2: 2 letters of the code word are the same, while the third is different from the two.

AAR AAP AAB

Number of possible ways $= \frac{3!}{2!} \times 3 = 9$

PPA PPR PPB

Number of possible ways $= \frac{3!}{2!} \times 3 = 9$

BBA BBR BBP

Number of possible ways $= \frac{3!}{2!} \times 3 = 9$

Case 3: All 3 letters of the code are different from one another.

Number of possible ways $= {}^4C_3 \times 3! = 24$

$\therefore$ Total possible number of 3-letter code words that can be formed $2 + 9 + 9 + 9 + 24 = 53$ (shown)