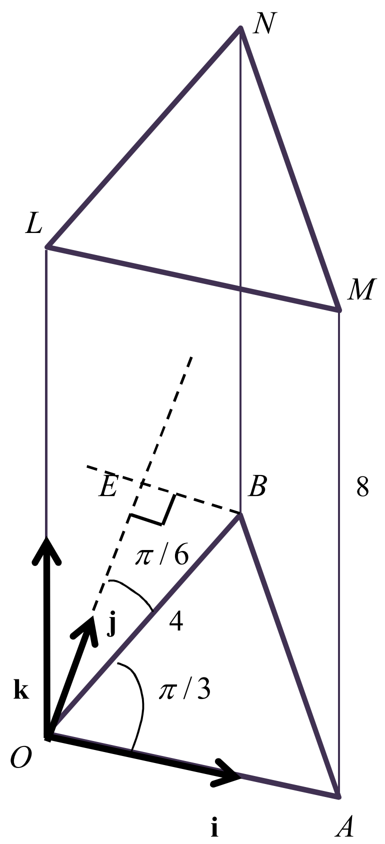
# Extreme Problem 9 Solutions




Let $E$ be a point such that it is coplanar with $OAB$ and triangle $OEB$ is a right angle triangle.

Then $OE = 4\cos\left(\frac{\pi}{6}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$ units and $BE = 4\sin\left(\frac{\pi}{6}\right) = 2$ units

Hence, $\overrightarrow{ON} = \begin{pmatrix} 2 \\ 2\sqrt{3} \\ 8 \end{pmatrix} // \begin{pmatrix} 1 \\ \sqrt{3} \\ 4 \end{pmatrix}$ and $\overrightarrow{OM} = \begin{pmatrix} 4 \\ 0 \\ 8 \end{pmatrix} // \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

A vector normal to plane $OMN = \begin{pmatrix} 1 \\ \sqrt{3} \\ 4 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2\sqrt{3} \\ 2 \\ -\sqrt{3} \end{pmatrix}$

$\therefore$ Equation of plane $OMN$ is $r \bullet \begin{pmatrix} 2\sqrt{3} \\ 2 \\ -\sqrt{3} \end{pmatrix} = 0 \Rightarrow 2\sqrt{3}x + 2y - \sqrt{3}z = 0$ (shown)