Extreme Problem 8 Solutions

$$y = \frac{x}{t^3} \Rightarrow yt^3 = x \text{ -------- (1)}$$

Differentiating both sides $wrt\ t$, $t^3 \frac{dy}{dt} + 3yt^2 = \frac{dx}{dt}$

$$t^2\left(t\frac{dy}{dt} + 3y\right) = \frac{dx}{dt} \text{ ---------- (2)}$$

$$t\frac{dy}{dt} - 3y(t-1) = \left(yt^2\right)^2 \Rightarrow \left(t\frac{dy}{dt} + 3y\right) - 3yt = \left(yt^2\right)^2$$

Multiplying both sides by t^2 gives

$$t^2\left(t\frac{dy}{dt} + 3y\right) - 3yt^3 = t^2\left(yt^2\right)^2 = \left(yt^3\right)^2$$

Using the above substitutions (1) and (2), $t^2\left(t\frac{dy}{dt} + 3y\right) - 3yt^3 = \left(yt^3\right)^2$ becomes $\frac{dx}{dt} - 3x = x^2$

$$\frac{dx}{dt} = x^2 + 3x$$

$$\int \frac{1}{x^2 + 3x} dx = \int dt$$

$$\int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}} dx = t + C$$

$$\frac{1}{2\left(\frac{3}{2}\right)} \ln\left|\frac{x + \frac{3}{2} - \frac{3}{2}}{x + \frac{3}{2} + \frac{3}{2}}\right| = t + C$$

$$\frac{1}{3}\ln\left|\frac{x}{x+3}\right| = t + C$$

$$\ln\left|\frac{x}{x+3}\right| = 3t + B \qquad \left(\because B = 3C\right)$$

$$\frac{x}{x+3} = e^{3t+B} = De^{3t} \qquad \left(\because D = e^B\right)$$

$$\frac{x+3}{x} = Ae^{-3t} \qquad \left(\because A = \frac{1}{D}\right)$$

$$1 + \frac{3}{x} = Ae^{-3t}$$

$$x = \frac{3}{\left(Ae^{-3t} - 1\right)}$$

Since $x = yt^3$, $\quad yt^3 = \dfrac{3}{\left(Ae^{-3t} - 1\right)} \Rightarrow y = \dfrac{3}{t^3\left(Ae^{-3t} - 1\right)}$ (shown)