Extreme Problem 5 Solutions

(i) Let $x = a + b - u$, then $dx = -du$

When $x = b$, $u = a$; when $x = a$, $u = b$

$$\therefore \int_a^b f(x)dx = \int_b^a f(a+b-u)(-du) = \int_a^b f(a+b-u)du$$

$$= \int_a^b f(a+b-x)dx \quad (\because x \text{ and } u \text{ are now interchangeable dummy variables}) \text{ (shown)}$$

(ii) For $\displaystyle\int_3^9 \frac{\ln|9-x|}{\ln|x-9|+\ln|x-3|}dx$, $a = 3$, $b = 9$

By the above result, this integral can be transformed into

$$\int_3^9 \frac{\ln|9-(12-x)|}{\ln|(12-x)-9|+\ln|(12-x)-3|}dx = \int_3^9 \frac{\ln|x-3|}{\ln|3-x|+\ln|9-x|}dx$$

Then $$2\int_3^9 \frac{\ln|9-x|}{\ln|x-9|+\ln|x-3|}dx = \int_3^9 \frac{\ln|9-x|}{\ln|x-9|+\ln|x-3|}dx + \int_3^9 \frac{\ln|9-x|}{\ln|x-9|+\ln|x-3|}dx$$

$$= \int_3^9 \frac{\ln|9-x|}{\ln|x-9|+\ln|x-3|}dx + \int_3^9 \frac{\ln|x-3|}{\ln|3-x|+\ln|9-x|}dx$$

$$= \int_3^9 \frac{\ln|x-9|}{\ln|x-9|+\ln|x-3|}dx + \int_3^9 \frac{\ln|x-3|}{\ln|x-3|+\ln|x-9|}dx$$

$$= \int_3^9 \frac{\ln|x-9|+\ln|x-3|}{\ln|x-9|+\ln|x-3|}dx = \int_3^9 dx = [x]_3^9 = 9 - 3 = 6$$

Hence, $$\int_3^9 \frac{\ln|9-x|}{\ln|x-9|+\ln|x-3|}dx = \frac{1}{2}(6) = 3 \text{ (shown)}$$