

Extreme Problem 3

(a) Prove that $r \times a = b$ where a and b are non-zero vectors, represents a line with direction vector

a and is such that a and b are mutually perpendicular. If $a = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$, find the

vector equation of the line $r \times a = b$ in the form $r = p + \lambda q$.

(b) Given that A is a point with position vector a lying on the plane Π having equation $r \cdot n = p$, and B is a point with position vector b not lying on the plane Π ,

Prove that the position vector of the foot of perpendicular from B to Π is $b + \frac{(a-b) \cdot n}{|n|^2} n$.

Deduce that the image of B when reflected in Π has position vector $b + \frac{2(a-b) \cdot n}{|n|^2} n$.