

Extreme Problem 3 Solutions

(a) Equation of line in parametric form is $r = p + \lambda a$, where a denotes the direction vector of the line and p the position vector of a point on the line.

Then $r \times a = (p + \lambda a) \times a$ (note this is achieved by adding a cross product on both sides)

$$r \times a = p \times a + \lambda(a \times a) = p \times a \text{ ----- (1) } [\because a \times a = \underline{\underline{0}}]$$

Since b is mutually perpendicular to a and $(p \times a)$ indeed yields a vector perpendicular to a ,

$$\text{ie } p \times a = b$$

hence, (1) becomes $r \times a = b$ (shown)

$$\text{Let } p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \text{ then } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -2y - 2z \\ 2x + z \\ 2x - y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

This system of equations represented by

$$-2y - 2z = 2$$

$$2x + z = 1$$

$$2x - y = 2$$

can be articulated in the form of an augmented matrix

$$\left(\begin{array}{ccc|c} 0 & -2 & -2 & 2 \\ 2 & 0 & 1 & 1 \\ 2 & -1 & 0 & 2 \end{array} \right);$$

$$\text{Solving gives an infinite set of solutions } \begin{pmatrix} \frac{1}{2} \\ 2 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{pmatrix}$$

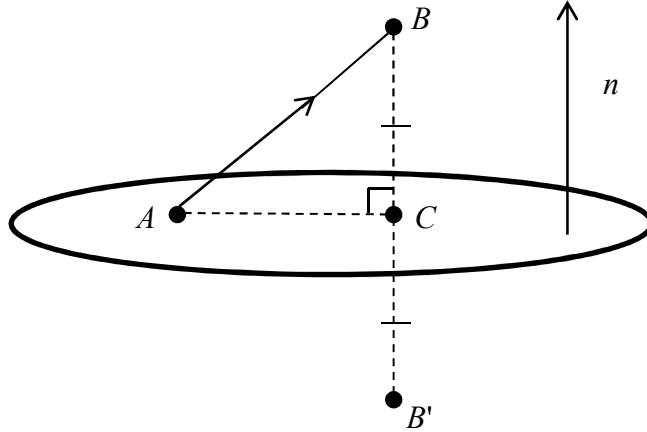
$$\text{Setting } \mu = -1, \text{ we have } x = 1, y = 0, z = -1 \Rightarrow p = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

(Note that we can use other values of μ ; the vector p would change accordingly. This is

allowed because different values of μ merely correspond to different points on the **same** line.)

$$\therefore \text{Equation of the line in parametric form is } r = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \text{ (shown)}$$

(b)



$$\vec{CB} = (\vec{AB} \cdot \hat{n}) \hat{n} = \left[(b-a) \cdot \frac{n}{|n|} \right] \frac{n}{|n|}$$

$$= \frac{(b-a) \cdot n}{|n|^2} n$$

$$\vec{OC} = \vec{OB} - \vec{CB} = b - \frac{(b-a) \cdot n}{|n|^2} n$$

$$= b + \frac{(a-b) \cdot n}{|n|^2} n \text{ (shown)}$$

$$\vec{OB'} = \vec{OB} + \vec{BB'} = \vec{OB} + 2\vec{BC'} = \vec{OB} - 2\vec{CB'}$$

$$= b - \frac{2(b-a) \cdot n}{|n|^2} n = b + \frac{2(a-b) \cdot n}{|n|^2} n \text{ (shown)}$$