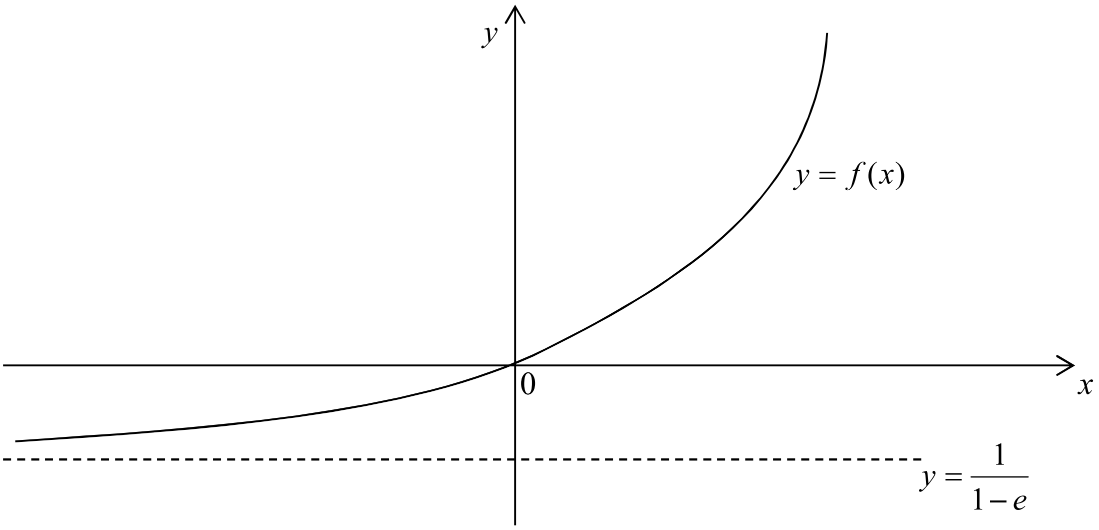
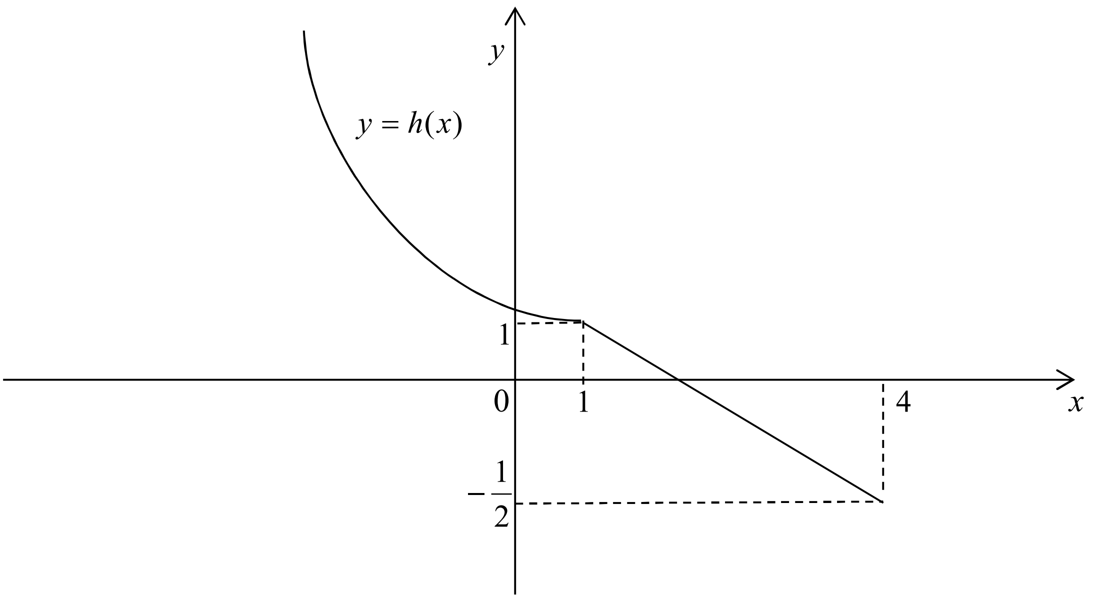
Extreme Problem 13 Solutions





To discover $h^{-1}x$:

<u>For $D_h = (-\infty,\ 1]$,</u>

Let $y = (x-1)^2 + 1 \Rightarrow -\sqrt{y-1} = x-1, \quad x = 1-\sqrt{y-1}$,

$\therefore \quad h^{-1}(x) = 1-\sqrt{x-1}$, $x \geq 1$ $\left(\because D_{h^{-1}} = R_h\right)$

<u>For $D_h = (1,\ 4]$,</u>

Let $y = 1 - \frac{|1-x|}{2} = 1 - \frac{-(1-x)}{2} = 1 + \frac{1-x}{2} = \frac{3}{2} - \frac{x}{2}$

Rearranging gives $x=3-2y$, $\therefore\ h^{-1}(x)=3-2x$, $-\frac{1}{2}\le x<1$ $\left(\because D_{h^{-1}}=R_h\right)$

$$\left(f^{-1}h\right)^{-1}(3)=h^{-1}f(3)=h^{-1}\left(\frac{e^3-1}{e-1}\right)=1-\sqrt{\left(\frac{e^3-1}{e-1}\right)-1}=1-\sqrt{\frac{e^3-1-e+1}{e-1}}$$

$$=1-\sqrt{\frac{e\left(e^2-1\right)}{e-1}}==1-\sqrt{\frac{e(e+1)(e-1)}{e-1}}=1-\sqrt{e(e+1)}\ \text{(shown)}$$