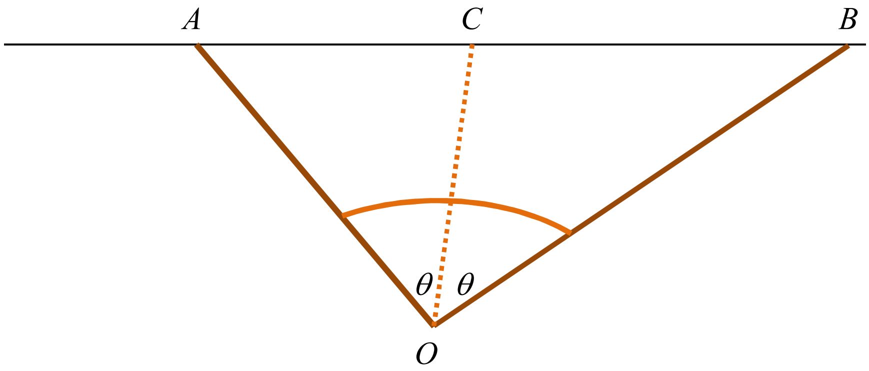
Given two points A and B in vector space, how does one discern a point on the line AB such that it bisects the angle AOB?



The premise is quite straight forward. In essence, it shall be demanded that this point C exists such that angle AOC equals angle BOC.

As such, $\cos\theta = \dfrac{a \bullet c}{|a||c|} = \dfrac{b \bullet c}{|b||c|} \Rightarrow \dfrac{a \bullet c}{|a|} = \dfrac{b \bullet c}{|b|}$ -------- (1)

Now, the equation of the line AB can be subsequently fashioned as $r = a + \lambda(b - a)$, where $\lambda \in \Re$

Thereafter, for a particular point C on the said line, this can also be interpreted as $c = a + \lambda(b - a)$ for a given value of λ presently unknown.

Substituting this into (1), we have $\dfrac{a \bullet [a + \lambda(b - a)]}{|a|} = \dfrac{b \bullet [a + \lambda(b - a)]}{|b|}$

Bearing in mind that the coordinates of both points A and B are already provided at the onset, one strives to solve for λ, which shall be re-inserted into $c = a + \lambda(b - a)$ to actually discover the point C itself.

Note: students may be tempted to perform a further reduction of (1) by simply construing $a \bullet b = b \bullet c$ as $a = b$ by eliminating c on both sides-this is absolutely incorrect. Consider a simple example, where $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} = 6$; clearly $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$