



The formula of a tetrahedron with vertices A , B , C and D is given by $\frac{1}{6} |(\vec{AB} \times \vec{AC}) \cdot \vec{AD}|$.

Here is the full derivation:

Let vector normal to plane ABC be n , then $n = \vec{AB} \times \vec{AC}$

Area of triangular base $ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} |n|$

Vertical height of vertex D above plane $ABC = l = |\vec{AD} \cdot \hat{n}| = \frac{1}{|n|} |\vec{AD} \cdot n|$

Volume of tetrahedron = $\frac{1}{3}$ (base area) (vertical height l)

$$\begin{aligned}
 &= \frac{1}{3} \left[\frac{1}{2} |n| \right] \left[\frac{1}{|n|} |\vec{AD} \cdot n| \right] \\
 &= \frac{1}{6} |\vec{AD} \cdot n| = \frac{1}{6} |(\vec{AB} \times \vec{AC}) \cdot \vec{AD}| \quad (\text{shown})
 \end{aligned}$$