How can we determine if two lines are exactly the same by inspecting their vector equations?

Here goes:

For two lines with equations $r = a + \lambda m$ and $r = b + \mu d$ where λ, μ are real valued parameters, and a, b represent position vectors of points lying on the lines, then

> If $m // d$ and b lies on $r = a + \lambda m$ for some value of λ, then the two lines are equivalent.
>
> (Alternatively, we can prove that a lies on $r = b + \mu d$ for some value of μ.)

Let's look at an example to facilitate understanding of the above requirements.

For $r = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$ and $r = \begin{pmatrix} 7 \\ -2 \\ 10 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -4 \end{pmatrix}$,

Since $\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} // \begin{pmatrix} -3 \\ 1 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ -2 \\ 10 \end{pmatrix}$ does lie on $r = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$ for $\lambda = 2$, hence both lines are **equivalent** to each other. (shown)