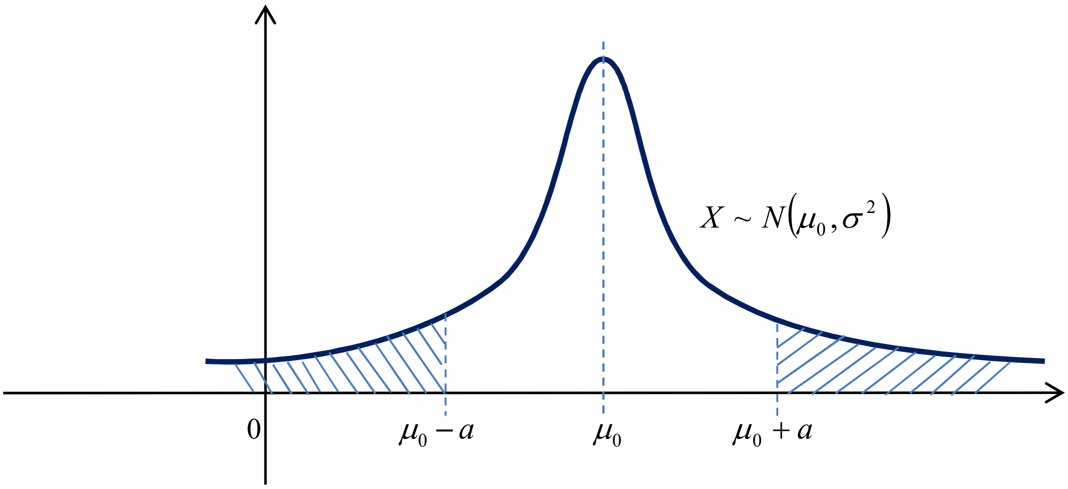
Say for instance a random variable X is normally distributed with a known mean $\mu = \mu_0$ and an unknown variance σ^2, and it is further given that $kP(X < \mu_0 - a) = P(X < \mu_0 + a)$, where both k and a are known, real positive constants, how would we go about solving for the value of σ efficiently?

The trick is to recognise some form of symmetry exists within the above problem.

Drawing the normal distribution curve in reference to the definition of rv X:



By observation, appreciate that the area (under the curve) to the left of $\mu_0 - a$ is **exactly equivalent to** the area (under the curve) to the right of $\mu_0 + a$, ie

$$P(X > \mu_0 + a) = P(X < \mu_0 - a) ------- (1)$$

Also, $P(X < \mu_0 + a) = 1 - P(X > \mu_0 + a)$

Substituting in (1), $P(X < \mu_0 + a) = 1 - P(X < \mu_0 - a)$

Thus $kP(X < \mu_0 - a) = P(X < \mu_0 + a)$ becomes

$$kP(X < \mu_0 - a) = 1 - P(X < \mu_0 - a)$$

$$(k+1) \bullet P(X < \mu_0 - a) = 1$$

$$P(X < \mu_0 - a) = \frac{1}{k+1}$$

Standardization to the standard normal Z distribution gives

$$P\left[Z < \frac{(\mu_0 - a) - \mu_0}{\sigma}\right] = \frac{1}{k+1} \qquad [\because Z \sim N(0,\ 1)\ ]$$

$$P\left(Z < -\frac{a}{\sigma}\right) = \frac{1}{k+1}$$

$$-\frac{a}{\sigma} = invNorm\left(\frac{1}{k+1}\right) \Rightarrow \sigma = -\frac{a}{invNorm\left(\frac{1}{k+1}\right)} \quad \text{(shown)}$$