

When solving for roots of higher degree polynomials in complex numbers, it is essential to convert  $e^{i\theta}$  into its equivalent form  $e^{i(2k\pi+\theta)}$  so that all roots can be discovered as the value of the **integer**  $k$  changes.

So how is  $e^{i(2k\pi+\theta)} = e^{i\theta}$  ?

I will provide the mathematical proof below, which is actually rather simple:

$$\begin{aligned} e^{i(2k\pi+\theta)} &= \cos(2k\pi + \theta) + i \sin(2k\pi + \theta) \\ &= \cos(2k\pi) \cos \theta - \sin(2k\pi) \sin \theta + i [\sin(2k\pi) \cos \theta + \cos(2k\pi) \sin \theta] \text{----- (1)} \end{aligned}$$

For all  $k \in \mathbb{Z}$ ,  $\cos(2k\pi) = 1$  and  $\sin(2k\pi) = 0$ ;

Hence (1) reduces to  $\cos \theta + i \sin \theta = e^{i\theta}$  (shown)

(Note: the trigonometric expansions  $\cos(A + B) = \cos A \cos B - \sin A \sin B$  and

$\sin(A + B) = \sin A \cos B + \cos A \sin B$  were employed in the above workings.)