In computing the unbiased estimate for the population variance, we encounter the following formula:

$$s^2 = \frac{1}{n-1}\left\{\sum (x-a)^2 - \frac{\left[\sum (x-a)\right]^2}{n}\right\}$$; however, when $a = \bar{x}$, this formula is simply reduced to

$$s^2 = \frac{1}{n-1}\sum (x-\bar{x})^2$$. Why is this so? Provided below is the mathematical proof:

$$\sum\left(x-\bar{x}\right) = \sum x - \sum \bar{x} = \sum x - \underbrace{\left(\bar{x}+\bar{x}+\bar{x}+\ldots\ldots\ldots.+\bar{x}\right)}_{n \text{ times}}$$

$$= \sum x - n\bar{x} = \sum x - n \bullet \frac{\sum x}{n} = \sum x - \sum x = 0$$

Therefore, when $a = \bar{x}$,

$$s^2 = \frac{1}{n-1}\left\{\sum (x-a)^2 - \frac{\left[\sum (x-a)\right]^2}{n}\right\}$$

$$= \frac{1}{n-1}\left\{\sum (x-\bar{x})^2 - \underbrace{\frac{\left[\sum (x-\bar{x})\right]^2}{n}}_{=0}\right\} = \frac{1}{n-1}\sum (x-\bar{x})^2 \text{ (shown)}$$