

For the DE of the form  $\frac{dy}{dx} + f(x) \cdot y = g(x)$ ,

We will multiply both sides by an integrating factor given by  $e^{\int f(x)dx}$ , such that the above becomes

$$e^{\int f(x)dx} \cdot \frac{dy}{dx} + \left[ f(x)e^{\int f(x)dx} \right] \cdot y = e^{\int f(x)dx} g(x) = h(x), \text{ where } h(x) = e^{\int f(x)dx} g(x)$$

This is equivalent through reduction (by the product rule for the LHS) to

$$\frac{d}{dx} \left[ ye^{\int f(x)dx} \right] = h(x)$$

Integrating both sides wrt  $x$  gives  $ye^{\int f(x)dx} = \int h(x)dx + C$

Therefore, the general solution is  $y = \left[ e^{-\int f(x)dx} \right] \cdot \left[ \int h(x)dx + C \right]$  (shown)