

The inverse rule for composite functions work as follows:  $(fg)^{-1} = g^{-1}f^{-1}$

When is this actually useful?

To find  $(fg)^{-1}$ , at times after obtaining the expression for  $fg$  it might be rather difficult (or seemingly impossible) to obtain its inverse. However, you can simply find the expressions for  $f^{-1}$  and  $g^{-1}$  separately, then subsequently embed  $g^{-1}$  within  $f^{-1}$  to give  $g^{-1}f^{-1}$ . This is exactly equivalent to  $(fg)^{-1}$ .

While such an application is less common in examination questions, nonetheless it would be good to have this knowledge. I have worked out the proof for the interested student below:

Let  $h = (fg)^{-1}$

Then  $h^{-1} = fg$

$$f^{-1}h^{-1} = f^{-1}(fg) = g \quad (\text{append } f^{-1} \text{ to the } \mathbf{front} \text{ of both sides of the equation})$$

$$g^{-1}f^{-1}h^{-1} = g^{-1}g = x \quad (\text{append } g^{-1} \text{ to the } \mathbf{front} \text{ of both sides of the equation})$$

$$\therefore (g^{-1}f^{-1}h^{-1})h = h(x) \Rightarrow h(x) = g^{-1}f^{-1} \text{ (shown)}$$

(append  $h$  to the **back** of both sides of the equation)