Statistics Revision 3 Solutions

1(i) Let X and Y be the random variables denoting the number of alphabetical and numerical misprints found in a book respectively.

Then $X \sim P_0(1)$ and $Y \sim P_0(2)$

Let T be the random variable denoting the total number of misprints found in a series of mathematics textbooks.

Then $T = X_1 + X_2 + ... + X_6 + Y_1 + Y_2 + ... + Y_6 \sim P_0(1 \times 6 + 2 \times 6 = 18)$

$P(T = 12) = 0.0368$ (shown)

(ii) $X + Y \sim P_0(1 + 2 = 3)$

$$P(X \geq 4 \mid X + Y = 5) = \frac{P(X = 4) \bullet P(Y = 1) + P(X = 5) \bullet P(Y = 0)}{P(X + Y = 5)}$$

$$= \frac{0.0153 \times 0.2707 + 0.0031 \times 0.1353}{0.1008} = 0.0452 \text{ (shown)}$$

2. Let X be the random variable denoting the length of a pair of boy's pants.

Then $X \sim N(\mu, \sigma^2)$

By symmetry $\mu = 85$ (since the normal distribution curve is symmetrical about the value when the area to both its left and right are equal to 0.5 exactly)

Also, $P(X < 80) = 0.2$

$$P\left(Z < \frac{80 - 85}{\sigma}\right) = 0.2$$

$$-\frac{5}{\sigma} = invNorm(0.2) = -0.8416$$

$$\therefore \ \sigma = \frac{-5}{-0.8416} = 5.9411 \text{ (shown)}$$

Let Y be the random variable denoting the length of a pair of girl's pants.

Then $Y \sim N(82,\ 5^2)$

$Y_1 + Y_2 \sim N(164,\ 5^2 + 5^2 = 50)$

$P(Y_1 + Y_2 \le 170) = 0.8019$ (shown)

Let X' and Y' be the random variables denoting the cost of a pair of boy's pants and a pair of girl's pants respectively.

Then $X' \sim N(0.35 \times 85 = 29.75,\ 0.35^2 \times 5.9411^2 = 4.3238)$

and $Y' \sim N(0.35 \times 82 = 28.7,\ 0.35^2 \times 5^2 = 3.0625)$

$X' - Y' \sim N(1.05,\ 7.3863)$

$\therefore\ P(X' - Y' > 0) = 0.6504$ (shown)

3(a) Let the mean and standard deviation of X be μ_x and σ_x respectively.

$Var(Y) = b^2 Var(X) = (b\sigma_x)^2 \Rightarrow$ standard deviation of $Y = b\sigma_x$

Since the standard deviation of Y is twice that of the standard deviation of X, $b = 2$ (shown)

$\mu_x + 0.8 = 7.92 \Rightarrow \mu_x = 7.12$

Also, $E(Y) = E(a + bX) = a + bE(X) = a + b\mu_x$

Since the mean of Y equals 7.92, then $a + b\mu_x = 7.92$, ie $a + 7.12b = 7.92$

Substituting $b = 2$, $a = 7.92 - 7.12(2) = -6.32$ (shown)

(b)(i) $R + S \sim N(3\mu,\ 2^2 + 3^2 = 13)$

$P(R + S > 1) = 0.9 \rightarrow P(R + S < 1) = 0.1$

$$P\left(Z < \frac{1 - 3\mu}{\sqrt{13}}\right) = 0.1$$

$$\frac{1 - 3\mu}{\sqrt{13}} = invNorm(0.1) = -1.2816$$

Hence, $\mu = \dfrac{1 + \sqrt{13}(1.2816)}{3} = 1.8736$ (shown)

(ii) $S - R \sim N(1.8736,\ 13)$

$P(S > R) = P(S - R > 0) = 0.6983$ (shown)

4(i) Let X be the random variable denoting the number of people who bought tickets but did not turn up for the flight.

Then $X \sim B\left(213, \frac{1}{50}\right)$

Since $np = \frac{213}{50} = 4.26 < 5$ and $p = \frac{1}{50} < 0.1$, $X \sim P_0(4.26)$ approximately.

$P($ More people arrive than there are seats available)

$= P(X < 213 - 210) = P(X < 3) = P(X \leq 2) = 0.2024$ (shown)

(ii) Let Y be the random variable denoting the number of people who bought tickets but did not turn up for the second flight.

Then $X \sim B\left(135, \frac{1}{75}\right)$

Since $np = \frac{135}{75} = 1.8 < 5$ and $p = \frac{1}{75} < 0.1$, $Y \sim P_0(1.8)$ approximately.

$X + Y \sim P_0(1.8 + 4.26 = 6.06)$ approximately

Hence, $P(X + Y = 5) = 0.1590$ (shown)

5(i) To test: $H_0 : \mu = 750$ against $H_1 : \mu < 750$

$\bar{x} = 746,\ n = 20,\ \sigma = 11$

By the Z test, test statistic $Z = -1.6262$, $p = 0.0520 > 0.04$

Hence, there is **insufficient** evidence at the 4% level to conclude that the population mean mass is less than 750g. (shown)

(ii) Let $\bar{X}$ be the random variable denoting the mean mass of a sample of size n.

Then $\bar{X} \sim N\left(750, \frac{11^2}{n}\right)$

$P(\bar{X} > 745) \geq 0.97 \rightarrow P(\bar{X} < 745) < 0.03$

$$P\left(Z < \frac{745-750}{\frac{11}{\sqrt{n}}}\right) < 0.03$$

$$\frac{-5}{\frac{11}{\sqrt{n}}} < invNorm(0.03) = -1.8808$$

$$\frac{-5\sqrt{n}}{11} < -1.8808 \Rightarrow \sqrt{n} > 4.1377 \text{ or } \quad n > 17.1209$$

$\therefore$ Smallest possible value of $n = 18$ (shown)

6(a)(i) Number of possible ways $= 4! = 24$ (shown)

(ii) For a 3 digit number 5 X 1, possible number of ways $= 2$ (X can either take 3 or 6)

For 5 X 3, possible number of ways $= 2$

For 6 X 1, possible number of ways $= 2$

For 6 X 3, possible number of ways $= 2$

For 6 X 5, possible number of ways $= 2$

For a 4 digit number ending with 1, possible number of ways $= 3! = 6$

For a 4 digit number ending with 3, possible number of ways $= 3! = 6$

For a 4 digit number ending with 5, possible number of ways $= 3! = 6$

Hence, total number of possible ways $= 2 \times 5 + 3 \times 6 = 28$ (shown)

(b) Consider the instances when both cards numbered 4 and 5 are next to each other. Treating them as a single unit, number of possible ways this can happen $= 5! \times 2! = 240$

Total number of ways to arrange the cards without restrictions $= 6! = 720$

$\therefore$ P (both cards numbered 4 and 5 are not next to each other) $= \frac{720-240}{720} = \frac{2}{3}$ (shown)

7(i) $X \sim N(125,\ 4.2^2 = 17.64)$

$P(X > 128) = 0.2375$ (shown)

(ii) $P(k < X < 128) = 0.7465 \rightarrow P(X < 128) - P(X < k) = 0.7465$

$(1 - 0.2375) - P(X < k) = 0.7465$

$P(X < k) = 0.016$

$$P\left(Z < \frac{k-125}{4.2}\right) = 0.016$$

$$\frac{k-125}{4.2} = invNorm(0.016) = -2.1444$$

$\therefore\ k = -2.1444(4.2) + 125 = 116$ (shown)

(iii) Let Y be the random variable denoting the number of bars of soap which weigh more than 128 grams each (out of a sample of 5)

Then $Y \sim B(5,\ 0.2375)$

$\therefore P(Y > 2) = 1 - P(Y \leq 2) = 1 - 0.9092 = 0.0908$ (shown)