Additional Revision Questions 4

1. A fund is started at \$1000 and compound interest is reckoned at 4% per annum (at the end of each year). If withdrawals of \$50 are made at the beginning of each of the subsequent years, show that the amount in the fund at the beginning of the $(n+1)th$ year is

$$1250\left[1-\frac{1}{5}\left(1.04\right)^{n}\right].$$

2. Find the following sums, giving your answers in terms of n:

(i) $\sum_{r=n+1}^{2n}(r-2n)^2$ (ii) $\sum_{r=12}^{n}\left(2^r-2^{r-1}\right)$

3. Show that $\frac{r^2+r-1}{(r+1)!}=\frac{1}{(r-1)!}-\frac{1}{(r+1)!}$.

Hence, work out $\sum_{r=0}^{n}\frac{r^2+r-1}{(r+1)!}$, presenting your answer in the form $A+\frac{f(n)}{(n+1)!}$, where A is an integer and $f(n)$ is a function in n.

4(i) By considering the identity $4\sin^3 A \equiv 3\sin A-\sin 3A$, show that

$$\sum_{r=0}^{n}\frac{1}{3^r}\sin^3\left(3^r\theta\right)=\frac{1}{4}\left[3\sin\theta-\frac{1}{3^n}\sin\left(3^{n+1}\theta\right)\right].$$

(ii) Hence, find the infinite sum of

$$\sin^3\left(\frac{\pi}{2}\right)+\frac{1}{3}\sin^3\left(\frac{3\pi}{2}\right)+\frac{1}{3^2}\sin^3\left(\frac{3^2\pi}{2}\right)+\frac{1}{3^3}\sin^3\left(\frac{3^3\pi}{2}\right)+\text{.........}$$

5. The points A and B are equidistant from the origin O and have position vectors a and b (referred to O) such that the acute angle AOB is $\frac{\pi}{4}$ radians. The point N on AB exists such that $AB:NB=1:2$ and the point M is the foot of perpendicular of N on OB.

(i) Show that the position vector of the point M is $\frac{1}{3}\left(\sqrt{2}+1\right) b$.

(ii) If it is further known that b is a unit vector, find the exact area of triangle OMN.

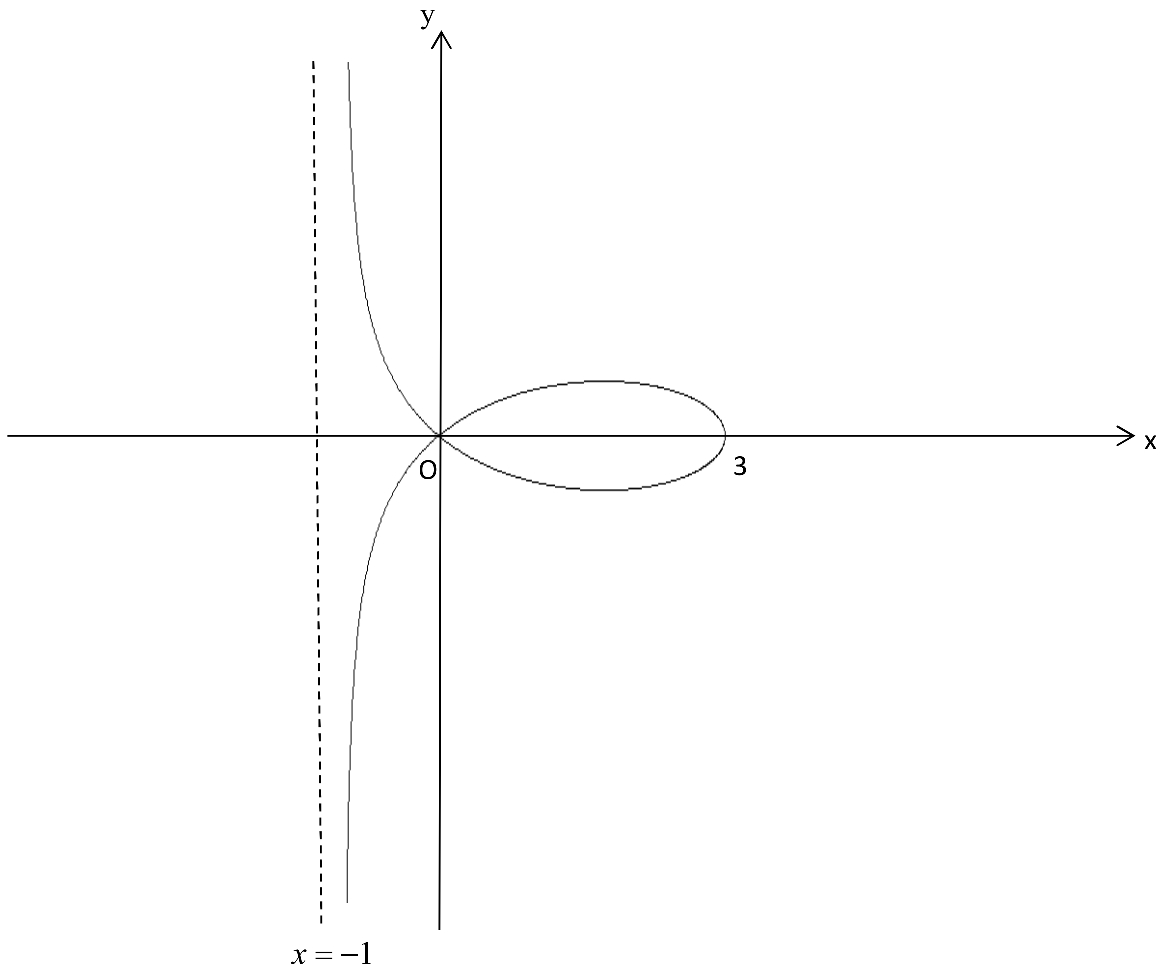
6(a) Find $\int \frac{8x^2+1}{4x^2-1} dx$.

(b) By considering integration by parts, find $\int e^{3x} \tan^{-1}\left(e^{-3x}\right) dx$.

7. The diagram below shows a curve C which is defined parametrically by

$x = 4\cos^2\theta - 1, \quad y = \left(4\cos^2\theta - 1\right)\tan\theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The curve intersects the

$x-$axis at the origin and at the point with coordinates $(3, 0)$.



(i) Show that $\frac{dy}{dx} = \frac{8\sin^2\theta + \sec^2\theta - 4}{8\sin\theta\cos\theta}$.

What can be said about the tangent to the curve C at $\theta = 0$?

(ii) Find the value of θ at the origin.

The region enclosed by C is denoted by R.

(iii) Find the exact area of R.

answer correct to 3 significant figures.

8. By using mathematical induction, prove that

(iv) Find the volume of revolution when R is revolved π radians about the $x-$axis. Give your

$$\sum_{r=1}^{n} \cos(2r\theta) = \frac{\sin[(2n+1)\theta] - \sin\theta}{2\sin\theta},\ 0 < \theta < \pi,$$

for all $n \in Z^+$. Hence find an expression for $\sum_{r=1}^{n} \cos^2(r\theta)$ in terms of n.

9.Find $\frac{d}{dx}\left[\cot\left(x^2\right)\right]$. By considering this result or otherwise, evaluate $\int x^3 \cos ec\left(x^2\right) dx$.

10 (i) Find the value of A such that $\int \left(\frac{e^x}{e^{2x}+1}\right)^2 dx = \frac{A}{e^{2x}+1} + C$, where C is an arbitrary real

constant.

(ii) The region bounded by the curve $y = \frac{e^x}{e^{2x}+1}$, the $x-$axis, the $y-$axis and the line $x = \ln 2$

is rotated 2π radians about the $x-$axis to form a solid. By considering the result obtained

in (i), find the exact volume of this solid.