<u>Additional Revision Questions 3</u>

1. The complex number $2+2i$ is denoted by w.

(i) Sketch an Argand diagram showing the points representing 1, i and w. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z-1| \le |z-i|$ and $|z-w| = 1$.

(ii) Using your diagram, calculate the value of $|z|$ for the point in this region such that $\arg(z)$ is a minimum.

2 (a) By considering the substitution $u = \tan x$, show that, for $n \neq 1$,

$$\int_0^{\frac{\pi}{4}} (\tan^{n+2} x + \tan^n x)dx = \frac{1}{n+1}.$$

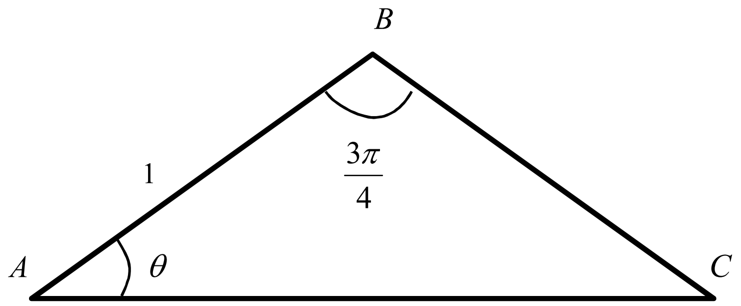
(b)Solve the differential equation $\frac{dy}{dx} = y(4-y)$, expressing y in terms of x.

3.The number of birds of a certain species in a de-forested area is recorded over a span of several years. At time t years, the number of birds is N, where N is a variable. The variation if the number of birds is modelled by $\frac{dN}{dt} = \frac{N(1800-N)}{3600}$. It is further given that the initial population is 300 birds.

(i) Find an expression for N in terms of t.

(ii) According to the model, how many birds will there be in the long run?

4.



In the above triangle ABC, $AB = 1$, angle $BAC = \theta$ radians and angle $ABC = \frac{3\pi}{4}$ radians.

(i) Show that $AC = \frac{1}{\cos\theta - \sin\theta}$.

(ii) Given that θ is sufficiently small, show that $AC \approx 1 + a\theta + b\theta^2$, where a and b are constants to be determined.

5. The curve C has equation $x - y = (x + y)^2$. It is also given that C has only one turning point.

(i) Show that $1 + \frac{dy}{dx} = \frac{2}{2x + 2y + 1}$.

(ii) Hence or otherwise, show that $\frac{d^2y}{dx^2} = -\left(1 + \frac{dy}{dx}\right)^3$.

(iii) Deduce whether this turning point is a maximum or a minimum.

6. The complex number z is given by $z = 1 + id$, where d is a non-zero real number.

Without the use of a calculator,

(i) Find z^3 in the form $x + iy$.

(ii) Given that z^3 is real, find the possible values of z.

(iii) For the value of z found in (ii) for which $d < 0$, find the smallest positive integer n such that $|z^n| > 1000$. State the modulus and argument of z^n when n assumes this value.

7. Mr Toh invested \$50, 000 in 3 funds A, B and C. Each fund is associated with different level of risk and therefore offers a different rate of return.

In 2012, the rates of return for funds A, B and C are 6%, 8% and 10% respectively. In total, Mr Toh received a total return of \$3700. He invested twice as much money in Fund A as in Fund C. How much did he invest in each of the funds in 2012?

8. A sequence u_1, u_2, u_3............. is given by $u_1 = 2$ and $u_{n+1} = \dfrac{3u_n - 1}{6}$ for $n \geq 1$.

(i) Find the exact values of u_2 and u_3.

(ii) It is given that $u_n \to l$ as $n \to \infty$. Showing your working, find the exact value of l.

(iii) For this value of l, prove by mathematical induction that $u_n = \dfrac{14}{3}\left(\dfrac{1}{2}\right)^n + l$.

9. By considering the substitution $u = 2x + y$, show that the differential equation

$\dfrac{dy}{dx} = \sin^2(2x + y) - 3$ can be reduced to $\dfrac{du}{dx} = -\cos^2 u$. Hence, given that the solution curve passes through the origin, find the particular solution in the form $y = f(x)$.

10. Given that $y = \ln(\cos 2x)$, show that $\dfrac{d^2y}{dx^2} = -4 - \left(\dfrac{dy}{dx}\right)^2$.

By further differentiation, find the first two non-zero terms in the Maclaurin's expansion of y in ascending powers of x.

Deduce the Maclaurin's expansion for $\ln(\sec^2 2x)$, up to and including the term in x^4.