Additional Revision Questions 2

1 . The lines l_1 and l_2 are given by the equations

$$r = \begin{pmatrix} -6 \\ -3 \\ - \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \text{ respectively.}$$

(i) Find the acute angle between l_1 and l_2.

The points P and R lie on l_1 and l_2 respectively such that R is the reflection of point P in the line l. The three lines intersect the point Q (0, 3, -6).

(ii) Find a possible pair of vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ such that angle PQR is acute and $|\overrightarrow{QP}| = 5$.

Hence, obtain the vector equation of line l.

The Cartesian equations of the planes $\prod_2$ and $\prod_3$ are given by

$\prod_2 :\ 16x + 11y - z = 39$

$\prod_3 :\ ax + 4y + z = b$, where a and b are constants.

(iii) Given that the plane $\prod_1$ contains the points P, Q and R, find the vector equation of $\prod_1$ in scalar product form.

(iv) Verify that the line l_2 lies in $\prod_2$.

(v) Therefore, find the values of a and b such that all 3 planes $\prod_1$, $\prod_2$ and $\prod_3$ have no point in common.

2(a) Given that $2 - 3i$ is a root of the equation $3z^3 + az^2 + 43z + b = 0$, where $a, b \in \Re$, find, in no particular order, the values of a and b, and the remaining two roots.

Explain how the points representing the equation $3iw^3 - aw^2 - 43iw + b = 0$, where a, $b \in \Re$, in an Argand diagram can be obtained those representing the roots of the equation in the previous part.

(b) Solve $\sqrt{2}(z-1)^4 = -1-i$, expressing your answer in the form $1+e^{\frac{i\pi}{16}(\alpha k-3)}$, for $k = \pm1,\ 0,\ 2$, where α is a constant to be determined. Show that the modulus of the one of the roots above is $2\cos\frac{5\pi}{32}$.

3. Solve the following integrals:

(a) $\int 3^{\sqrt{2x+1}}\,dx$ (b) $\int \frac{x}{\sqrt{1-x^4}}\left[\sin^{-1}\left(x^2\right)\right]^3 dx$

4 (a) Obtain the derivative of $\ln\left(\tan^3 2x\right)$, expressing your answer as a single trigonometric function.

Hence, find $\int \frac{\ln\left(\tan^3 2x\right)}{\sin 4x}dx.$

(b) State, by observation, the smallest positive value of α such that $\int_0^{\alpha} \sin 2x \sin^2\left(\cos^2 x\right)dx = 0$.

5. WG News Corporation has 150 printing presses. The probability of a printing press requiring minor repair weekly is p. The number of printing presses requiring minor repair weekly is denoted by the variable X. If it is known that $75Var(X) = 2[E(X)]^2$, solve for p.

(i) Find the probability that less than 28 printing presses require minor repair in a week.

(ii) Find the minimum value of n such that the probability of at most n printing presses requiring minor repair weekly exceeds 0.6.

6. An orphanage received a donation of 100 buns, of which 10 buns contained neither cheese nor ham. Of the remaining buns, 60 contained cheese and x contained ham. The events C and H are defined as follows:

C : A randomly chosen bun contained cheese.

H : A randomly chosen bun contained ham.

(i) The conditional probability that a randomly chosen bun contained ham given that it contained cheese is 0.75. Find the value of x, and hence determine if C and H are independent events.

(ii) A child selected three buns at random. Find the probability that two of them contained only cheese while one contained both cheese and ham.

(iii) 45% of the buns donated were contaminated and would cause severe stomach pains if consumed. Sixty children unknowingly consumed a bun each and 30% of them suffered such pains. Find the conditional probability that among the buns which were not consumed, a randomly chosen bun was contaminated.

7. The complex number z satisfies the following relations:

$$|z-1-i| = \left|\sqrt{3}-i\right| \quad \text{and} \quad 0 \le \arg(z-5+3i) \le \frac{3\pi}{4}.$$

(i) On a single Argand diagram, illustrate both of these relations.

(ii) Find the exact values of z which give $\arg(z)_{\max}$ and $\arg(z)_{\min}$.

8. Let α denote the angle between unit vectors a and b, where $0 \le \alpha \le \pi$.

(i) Express $|a-b|$ and $|a+b|$ in terms of α.

(ii) Hence solve for the value of α for which $|a+b| = 3|a-b|$.

9. In triangle ABC, angle A $= \dfrac{\pi}{3} + 3\alpha$, angle B $= \dfrac{\pi}{3} - \alpha$, AC $= 1$ and BC $= x$. If α is small enough such that α^3 and higher powers of α can be ignored, show that the shortest distance from A to BC is given by $\dfrac{\sqrt{3}}{2} - \alpha - \sqrt{3}\alpha^2$. However, if α is small enough such that α^2 and higher powers of α can be ignored, show that $3x = 3 + 4\sqrt{3}\alpha$.