

## Additional Complex Number Problems 2

1. A variable complex number  $z$  is described by  $|z| = 3$ , while a fixed point  $c$  is represented by the complex number  $4 + i$ . By plotting these on an Argand diagram, find the minimum and maximum values of  $|z + c|$ . If another complex number  $w$  is described the loci  $|w - 8| = 1$ , plot  $w$  on the same diagram and hence determine the minimum and maximum values of  $|w - z|$ .
  
- 2(i) Show that the variable locus  $z = (1 + \sqrt{2} \cos \theta) + i(-1 + \sqrt{2} \sin \theta)$  can be described by a circle of radius  $\sqrt{2}$  units centered at  $1 - i$ .
  
- (ii) Represent in an Argand diagram, the region represented by  $\frac{\pi}{6} < \arg\left(\frac{5}{w}\right) < \frac{\pi}{2}$ .
  
- 3 (i) If  $z = \cos \theta + i \sin \theta$ , prove that  $\frac{z^2 - 1}{z^2 + 1} = i \tan \theta$ .
  
- (ii) Let  $z$  and  $w$  represent complex numbers such that  $|z|^2 = 1$  and  $w = \frac{1}{1 - z}$ .  
  
Find the real part of  $w$ .
  
4. Solve  $z^6 = -1 - i$ . Hence by considering this result, write down the roots of the equation  $w^6 = 1 + i$ .
  
- 5(i) If a variable complex number  $z$  exists such that  $|z| \geq |z - 4i|$ , show that  $\text{Im}(z) \geq 2$ .
  
- (ii) It is also known that  $z$  satisfies the relation  $|z - 1| = 4$ . On an Argand diagram, show the feasible region which  $z$  can reside within. Find the range of values  $\arg(z - 1)$  can assume.
  
- (iii) The loci  $|z| = |z - 4i|$  intersects  $|z - 1| = 4$  at a point such that the argument of the complex

number represented by the said point has an argument which lies between  $0$  rad and  $\frac{\pi}{2}$  rad as referenced from the origin. Find this complex number, giving your answer in exact form.

6 (i) A complex number  $w$  has modulus  $1$  and argument  $\theta$ , where  $0 < \theta < \frac{\pi}{2}$ . Given that  $p = \frac{w}{w^*}$ ,

State the values of  $|p|$  and  $\arg(p)$ . If it is known that  $p^5$  is purely imaginary, find the possible values of  $\theta$ .

(ii) A complex number  $z$  is defined by the loci  $|z| \leq \sqrt{3}$  and  $\text{Im}(z) > 0$ . Find the range of values of  $\arg(z - 2)$ .