Additional Calculus Problems 2 (Integration And Applications)

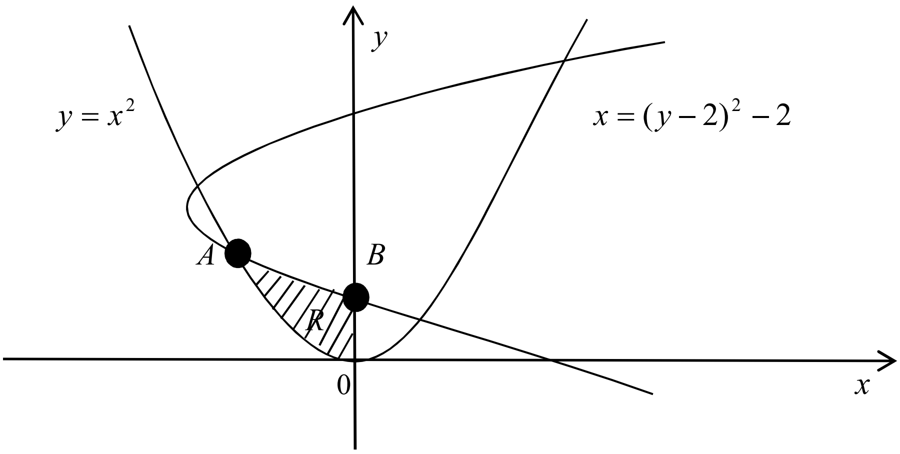
1(a) Using the substitution $u = \frac{1}{x}$, evaluate the exact value of $\int_{\frac{1}{2}}^{1} \frac{1}{x(5x^2 + 2x + 1)} dx$.

(b) Find $\int (x - e^{\cos x}) \sin x \, dx$.

2(a) A curve C is defined by the parametric equations $x = t - 1$, $y = \frac{30}{t^2 - 1}$, where $t > 1$.

Find the area bounded by the curve, the y-axis and the lines $y = 2$ and $y = 10$, giving your answer in exact form.

(b) The diagram below shows the region R bounded by the two parabolas $y = x^2$ and $x = (y-2)^2 - 2$ and the y-axis. Find the points indicated A and B in the diagram. Find the volume formed when R is rotated 2π radians about the y-axis.



3. Find $\frac{d}{dx} \ln(\ln x^{2x})$, where $-1 < x < 0$.

Hence, find $\int \ln(\ln x^{2x}) \frac{2 + \ln(x^2)}{x \ln(x^2)} dx$, where $-1 < x < 0$.

4. The region R is bounded by the curve $y = \frac{x}{1 + x^2}$, the line $x = 1$ and the $x-$axis. By means of the

substitution $x = \tan\theta$ or otherwise, find the exact value of $\int_0^1 \frac{x^2}{(1+x^2)^2}dx$.

Find the exact value of a and b such that the volume of revolution formed when R is rotated

completely about the line $y = \frac{1}{2}$ is $\frac{\pi}{a} - \frac{\pi^2}{8} + \frac{\pi}{2}\ln b$.

5(a) Find $\int \frac{5-2x}{\sqrt{2-4x-x^2}}dx$.

(b) Find $\int_0^N x^2 e^{-x} dx$. Hence find $\lim_{N\to\infty} \int_0^N x^2 e^{-x} dx$.

(c) Using the substitution $u = x+4$, evaluate exactly $\int_{-4}^{5} |x|\sqrt{x+4}\ dx$.

6. Show that, for any integer m,

$$\int_0^{2\pi} e^x \cos mx\ dx = \frac{1}{m^2+1}\left(e^{2\pi}-1\right).$$

Expand $\cos(A+B)+\cos(A-B)$. Hence show that

$$\int_0^{2\pi} e^x \cos x \cos 6x\ dx = \frac{19}{650}\left(e^{2\pi}-1\right).$$

7. Sketch the graph defined parametrically by $x = t^3 - 1,\ \ y = 5t^2 - 4,\ -2 \le t \le 2$. Find the area bounded by the curve, the positive axes and the line $x = 7$.