Additional Calculus Problems (Integration and Applications)

1. Evaluate the following integrals:

(a) $\int \frac{1+x}{1-x}dx$ (b) $\int \frac{1}{\sqrt{x}-\sqrt{x+1}}dx$ (c) $\int (e^t + \frac{2}{e^t})^2 dt$

(d) $\int \frac{1}{e^{2t}-1}dt$ (e) $\int \sin\theta(1+\cos\theta)d\theta$ (f) $\int \frac{12}{4t^2+8t-5}dt$

(g) $\int \frac{2+t^2}{(1-t)^2}dt$ (h) $\int \theta(\cos\theta+\sin\theta)^2 d\theta$ (i) $\int \ln\left(\frac{1}{1+x}\right)dx$

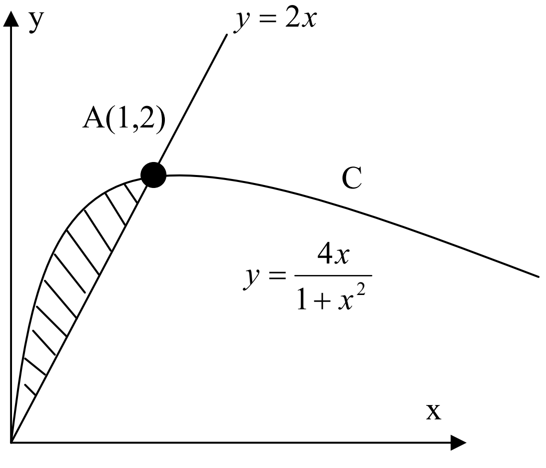
2. By means of the substitution $u = 2\sqrt{x}+1$, or otherwise, find the exact value of

$$\int_0^4 \frac{2}{2\sqrt{x}+1}dx$$

3. The diagram shows the region R bounded by the curve C with equation $y = \frac{4x}{1+x^2}$ and the lines $y = 2x$.
The point of intersection of the two graphs, A(1,2) is also a turning point of C.

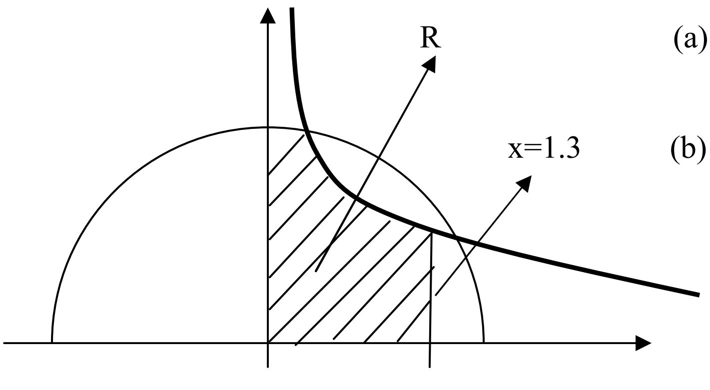
(a) Find the area of R.

(b) Using the substitution $x = \tan\theta$, where appropriate, find the volume of the solid formed when R is rotated through four right angles about the x-axis.



4. (i) By using the substitution $x = 2\sin\theta$, find the exact value of $\int_0^1 \sqrt{\frac{1}{3}(4-x^2)}dx$.

(ii) The region R, bounded by the curves $y = \frac{1}{x}$ and $y = \sqrt{\frac{1}{3}(4-x^2)}$, the x-axis, the y-axis and the lines x=1.3 is shown in the diagram below:



(a) Find the area of R, correct to 3 significant figures.

(b) Find the volume formed when R is rotated through 4 right angles about the x-axis.

5. (a) Evaluate $\int_{-2}^{2} xe^{|x|}dx$.

(b) Find $\int \frac{-8x-3}{\sqrt{1-(2x+1)^2}}dx$.

6. (i) Show, using integration by parts, that

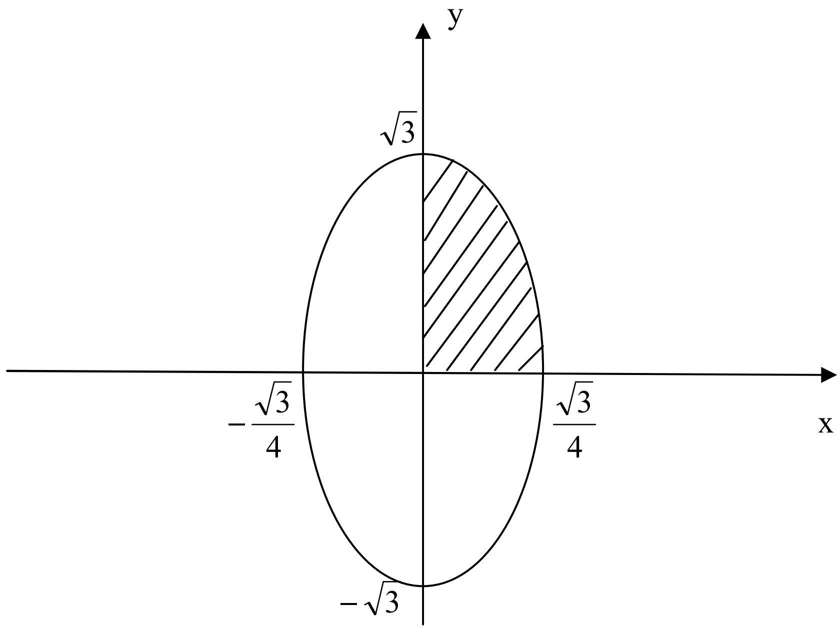
$$\int x^n \ln x dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + c.$$

(ii) Hence, express $\frac{d}{dx}[x^{100}(\ln x^{100} - 1)]$ in the form $x^a \ln x^b$, where a and b are constants to be determined.

(iii) Write down an expression for $\int x^n \ln x^n dx.$

7. Sketch, on the same diagram, the graphs of $y = x+2$ and $y = \frac{2}{x}+1$. Indicate, on your diagram, the region bounded by the lines $x = 4$, $y = 1$, $y = x+2$, the y-axis and the curve $y = \frac{2}{x}+1$. Hence find the exact area of this region.

8.



The equation $16x^2 + y^2 = 3$ represents the ellipse shown in the diagram.

(i) Show that the area of the shaded region, A, is given by

$$\int_0^{\frac{\sqrt{3}}{4}} \sqrt{3-16x^2}\,dx.$$

(ii) Using the substitution $x = \frac{\sqrt{3}}{4}\sin\theta$, find the exact value of A.

(iii) Find the exact volume of the solid obtained when the shaded region is rotated 2π radians about the y-axis.