

Extreme Problem 5 Solutions

(i) Let $x = a + b - u$, then $dx = -du$

When $x = b$, $u = a$; when $x = a$, $u = b$

$$\therefore \int_a^b f(x)dx = \int_b^a f(a+b-u)(-du) = \int_a^b f(a+b-u)du$$

$$= \int_a^b f(a+b-x)dx \quad (\because x \text{ and } u \text{ are now interchangeable dummy variables) \text{ (shown)}$$

(ii) For $\int_3^9 \frac{\ln |9-x|}{\ln |x-9| + \ln |x-3|} dx$, $a = 3$, $b = 9$

By the above result, this integral can be transformed into

$$\int_3^9 \frac{\ln |9-(12-x)|}{\ln |(12-x)-9| + \ln |(12-x)-3|} dx = \int_3^9 \frac{\ln |x-3|}{\ln |3-x| + \ln |9-x|} dx$$

$$\begin{aligned} \text{Then } 2 \int_3^9 \frac{\ln |9-x|}{\ln |x-9| + \ln |x-3|} dx &= \int_3^9 \frac{\ln |9-x|}{\ln |x-9| + \ln |x-3|} dx + \int_3^9 \frac{\ln |9-x|}{\ln |x-9| + \ln |x-3|} dx \\ &= \int_3^9 \frac{\ln |9-x|}{\ln |x-9| + \ln |x-3|} dx + \int_3^9 \frac{\ln |x-3|}{\ln |3-x| + \ln |9-x|} dx \\ &= \int_3^9 \frac{\ln |x-9|}{\ln |x-9| + \ln |x-3|} dx + \int_3^9 \frac{\ln |x-3|}{\ln |x-3| + \ln |x-9|} dx \\ &= \int_3^9 \frac{\ln |x-9| + \ln |x-3|}{\ln |x-9| + \ln |x-3|} dx = \int_3^9 dx = [x]_3^9 = 9 - 3 = 6 \end{aligned}$$

$$\text{Hence, } \int_3^9 \frac{\ln |9-x|}{\ln |x-9| + \ln |x-3|} dx = \frac{1}{2}(6) = 3 \text{ (shown)}$$