Extreme Problem 3 Solutions

(a) Equation of line in parametric form is $r = p + \lambda a$, where a denotes the direction vector of the line and p the position vector of a point on the line.

Then $r \times a = (p + \lambda a) \times a$ (note this is achieved by adding a cross product on both sides)

$r \times a = p \times a + \lambda(a \times a) = p \times a$ - - - - - - - (1) $[\ \because a \times a = \underset{\sim}{0}\ ]$

Since b is mutually perpendicular to a and $(p \times a)$ indeed yields a vector perpendicular to a,

$$\text{ie} \quad p \times a = b$$

hence, (1) becomes $r \times a = b$ (shown)

Let $p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, then $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -2y-2z \\ 2x+z \\ 2x-y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

This system of equations represented by

$$-2y - 2z = 2$$

$$2x + z = 1$$

$$2x - y = 2$$

can be articulated in the form of an augmented matrix

$$\left(\begin{array}{ccc|c} 0 & -2 & -2 & 2 \\ 2 & 0 & 1 & 1 \\ 2 & -1 & 0 & 2 \end{array}\right);$$

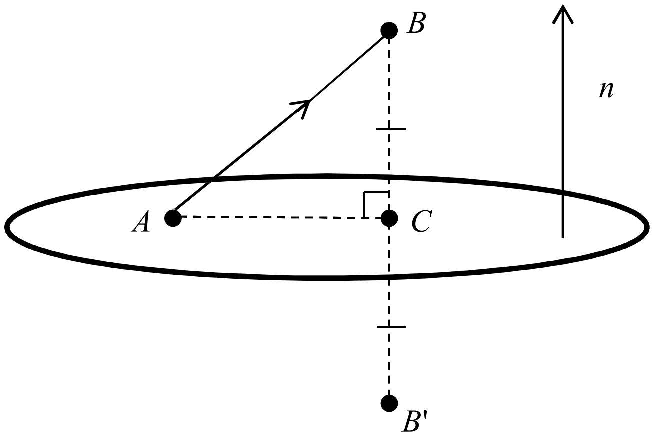
Solving gives an infinite set of solutions $\begin{pmatrix} \frac{1}{2} \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{pmatrix}$

Setting $\mu = -1$, we have $x = 1,\ y = 0,\ z = -1 \Rightarrow p = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(Note that we can use other values of μ; the vector p would change accordingly. This is allowed because different values of μ merely correspond to different points on the **same** line.)

$\therefore$ Equation of the line in parametric form is $r = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ (shown)

(b)



$$\overrightarrow{CB} = \left(\overrightarrow{AB} \bullet \hat{n}\right) \hat{n} = \left[(b-a) \bullet \frac{n}{|n|}\right] \frac{n}{|n|}$$

$$= \frac{(b-a) \bullet n}{|n|^2} n$$

$$\overrightarrow{OC} = \overrightarrow{OB} - \overrightarrow{CB} = b - \frac{(b-a) \bullet n}{|n|^2} n$$

$$= b + \frac{(a-b) \bullet n}{|n|^2} n \quad \text{(shown)}$$

$$\overrightarrow{OB'} = \overrightarrow{OB} + \overrightarrow{BB'} = \overrightarrow{OB} + 2\overrightarrow{BC'} = \overrightarrow{OB} - 2\overrightarrow{CB'}$$

$$= b - \frac{2(b-a) \bullet n}{|n|^2} n \ = b + \frac{2(a-b) \bullet n}{|n|^2} n \quad \text{(shown)}$$