

Extreme Problem 2

The position vectors, relative to an origin O , of four points A , B , C and D are

$4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $5\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$, $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ respectively.

Show that a vector normal to the plane BCD is $\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ and find a Cartesian equation of this plane and the distance of A from this plane.

Show also that AB , AC and AD are of equal length and are perpendicular to one another.

Find the position vector of the point E which is distinct from A and is such that

$$EB = EC = ED = AB = AC = AD.$$