

Extreme Problem 2 Solutions

$$\begin{aligned} \text{A vector normal to plane } BCD &= \vec{BC} \times \vec{BD} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \\ 4 \end{bmatrix} \times \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \\ 4 \end{bmatrix} \\ &= \begin{pmatrix} -3 \\ -3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 15 \end{pmatrix} // \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \text{ (shown)} \end{aligned}$$

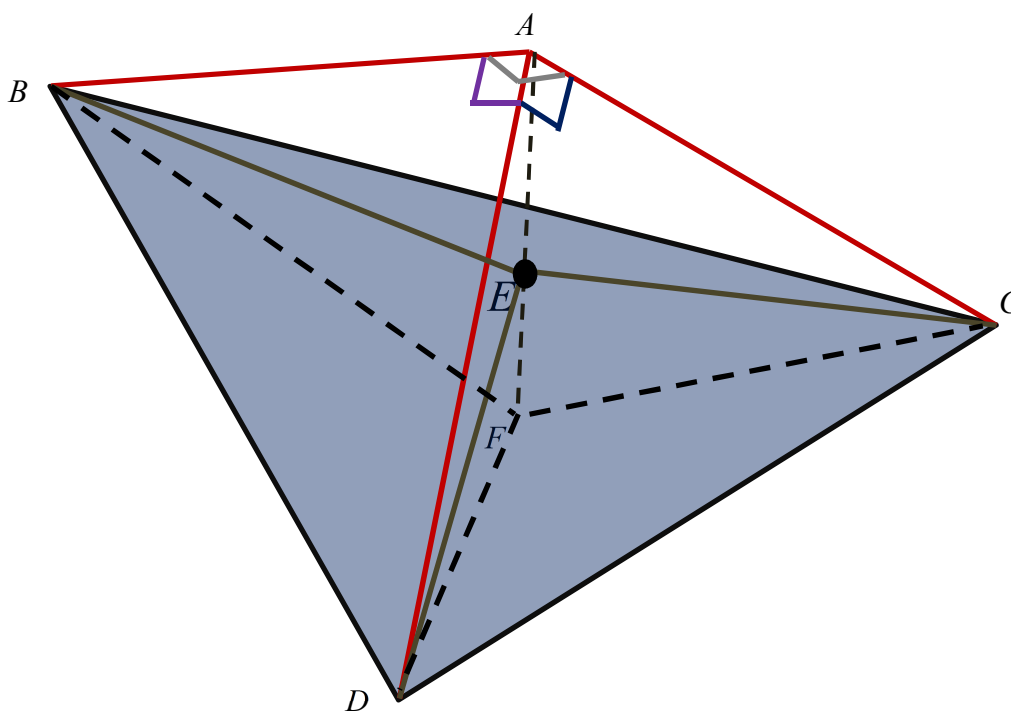
Equation of plane BCD is $x - y + z = (1)(2) + (-1)(3) + (5)(4) = 19$ (shown)

$$\text{Distance of } A \text{ from the plane} = |\vec{BA} \cdot \hat{n}| = \frac{1}{\sqrt{27}} \begin{vmatrix} -1 \\ -2 \\ -2 \end{vmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} = \frac{9}{\sqrt{27}} = \frac{9}{3\sqrt{3}} = \sqrt{3} \text{ units (shown)}$$

$$\vec{AB} = \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \quad \vec{AC} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}, \quad \vec{AD} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

Clearly, $|\vec{AB}| = |\vec{AC}| = |\vec{AD}| = 3$ units and $\vec{AB} \cdot \vec{AC} = \vec{AC} \cdot \vec{AD} = 0$

Hence, AB , AC and AD are of equal length and are perpendicular to one another. (shown)



From the above figure, point E is located along the normal to the plane BCD **but resides above** the foot of perpendicular from A to the plane BCD (designated point F)

$$\vec{OE} - \vec{OA} = k \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \Rightarrow \vec{OE} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} + k \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 4+k \\ 4-k \\ 2+5k \end{pmatrix}$$

$$|\vec{EB}| = 3 \Rightarrow \left| \begin{pmatrix} 4+k \\ 4-k \\ 2+5k \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} \right| = 3 \quad (*)$$

$$\left| \begin{pmatrix} k-1 \\ -2-k \\ 5k-2 \end{pmatrix} \right| = 3$$

$$\left(\begin{pmatrix} k-1 \\ -2-k \\ 5k-2 \end{pmatrix} \right)^2 = 9$$

$$(k-1)^2 + (-2-k)^2 + (5k-2)^2 = 9$$

$$k^2 - 2k + 1 + k^2 + 4k + 4 + 25k^2 - 20k + 4 = 9$$

$$27k^2 - 18k = 0 \rightarrow 3k(3k-2) = 0$$

$$k = 0 \text{ (rejected) or } k = \frac{2}{3}$$

$$\vec{OE} = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 14 \\ 10 \\ 16 \end{pmatrix} \quad \text{(shown)}$$

(*) Note: we can also solve for k by setting $|\vec{EC}| = 3$ or $|\vec{ED}| = 3$