# Extreme Problem 10 Solutions

Since $f(r) > f(r+1)$, then $f\left(k^{n+1}-1\right) > f\left(k^{n+1}\right)$, which implies the following is true:

$$
\begin{aligned}
&f\left(k^{n}\right) > f\left(k^{n+1}\right)\\
&f\left(k^{n}+1\right) > f\left(k^{n+1}\right)\\
&f\left(k^{n}+2\right) > f\left(k^{n+1}\right)\\
&\quad\vdots\\
&f\left(k^{n+1}-3\right) > f\left(k^{n+1}\right)\\
&f\left(k^{n+1}-2\right) > f\left(k^{n+1}\right)\\
&f\left(k^{n+1}-1\right) > f\left(k^{n+1}\right)
\end{aligned}
$$

As such, $f\left(k^{n}\right)+f\left(k^{n}+1\right)+f\left(k^{n}+2\right)+\ldots\ldots\ldots\ldots+f\left(k^{n+1}-3\right)+f\left(k^{n+1}-2\right)+f\left(k^{n+1}-1\right)$

$$= \sum_{r=k^{n}}^{k^{n+1}-1} f(r) > \underbrace{f\left(k^{n+1}\right)+f\left(k^{n+1}\right)+f\left(k^{n+1}\right)+\ldots\ldots\ldots\ldots+f\left(k^{n+1}\right)}_{k^{n+1}-k^{n}\,=\,k^{n}(k-1)\ \text{terms}}$$

Hence, $\displaystyle\sum_{r=k^{n}}^{k^{n+1}-1} f(r) > k^{n}(k-1)f\left(k^{n+1}\right)$ ------------------(1)

Also,

$$
\begin{aligned}
f\left(k^n\right) &= f\left(k^n\right) \\
f\left(k^n+1\right) &< f\left(k^n\right) \\
f\left(k^n+2\right) &< f\left(k^n\right) \\
&\vdots \\
f\left(k^{n+1}-1\right) &< f\left(k^n\right) \\
f\left(k^{n+1}-2\right) &< f\left(k^n\right) \\
f\left(k^{n+1}-1\right) &< f\left(k^n\right)
\end{aligned}
$$

As such, $f\left(k^n\right)+f\left(k^n+1\right)+f\left(k^n+2\right)+\ldots\ldots\ldots\ldots+f\left(k^{n+1}-3\right)+f\left(k^{n+1}-2\right)+f\left(k^{n+1}-1\right)$

$$= \sum_{r=k^n}^{k^{n+1}-1} f(r) < \underbrace{f\left(k^n\right)+f\left(k^n\right)+f\left(k^n\right)+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+f\left(k^n\right)}_{k^{n+1}-k^n = k^n(k-1) \text{ terms}}$$

Hence, $\displaystyle\sum_{r=k^n}^{k^{n+1}-1} f(r) < k^n(k-1)f\left(k^n\right)$ --------------- (2)

Reconciling (1) and (2) therefore gives $\displaystyle k^n(n-1)f\left(k^{n+1}\right) < \sum_{r=k^n}^{k^{n+1}-1} f(r) < k^n(n-1)f\left(k^n\right)$ (shown)