<u>Extreme Problem 1</u>

Relative to an origin O, the points A and B have position vectors a and b respectively, where

$a \bullet b > 0$. S is the sphere on which AB is the diameter and point C being the centre of this sphere.

Prove that tangents can be drawn from O to S and show that the length of each of these tangents

is $(a \bullet b)^{\frac{1}{2}}$. Show that each tangent makes an angle θ with OC, where

$$\cos\theta = \frac{2(a \bullet b)^{\frac{1}{2}}}{|\, a + b \,|}.$$

The cone formed by all tangents from O to S meets S in a circle of radius r.

Prove that $|\, a + b \,|^2 \; r^2 = (a \bullet b)\,|\, a - b \,|^2$.