Extreme Problem 12 Solutions

$$z_1 - z_2 = 2e^{(i)\left(\frac{\pi}{5}\right)} - 2e^{(i)\left(\frac{3\pi}{5}\right)}$$

$$= 2\left[\cos\left(\frac{\pi}{5}\right) + i\sin\left(\frac{\pi}{5}\right)\right] - 2\left[\cos\left(\frac{3\pi}{5}\right) + i\sin\left(\frac{3\pi}{5}\right)\right]$$

$$= 2\left[\cos\left(\frac{\pi}{5}\right) - \cos\left(\frac{3\pi}{5}\right)\right] - 2i\left[\sin\left(\frac{3\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right)\right]$$

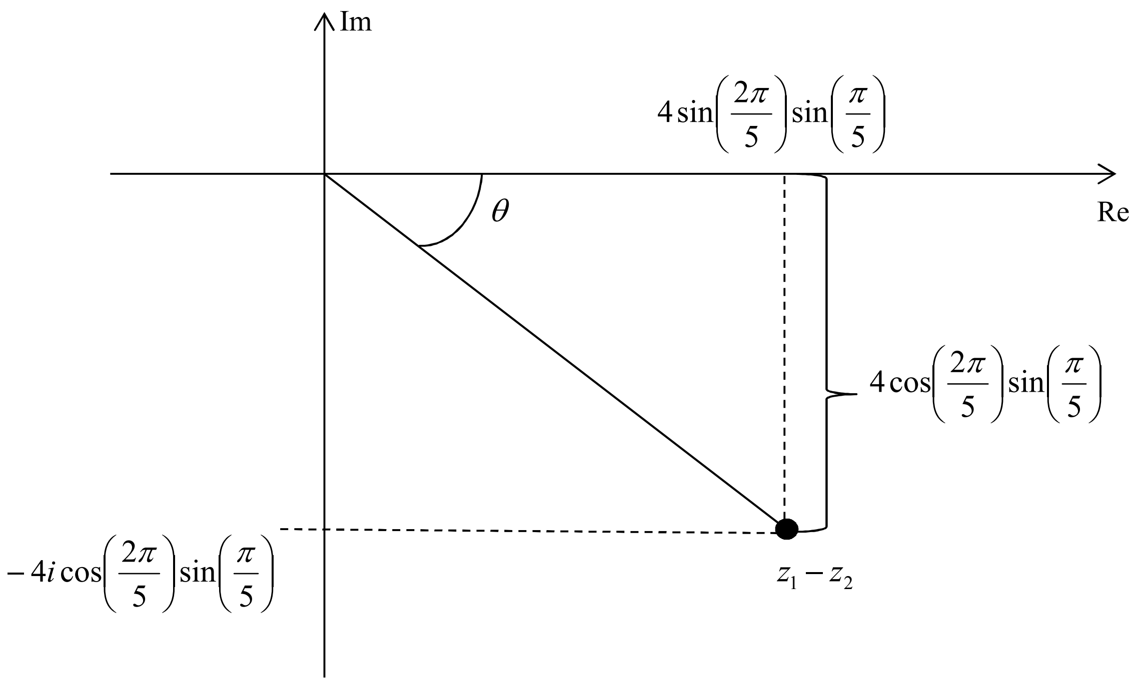
$$= 2\left[-2\sin\left(\frac{2\pi}{5}\right)\sin\left(-\frac{\pi}{5}\right)\right] - 2i\left[2\cos\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right)\right]$$

$$= 4\sin\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right) - 4i\cos\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right)$$

$$|z_1 - z_2| = \sqrt{\left[4\sin\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right)\right]^2 + \left[-4\cos\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right)\right]^2}$$

$$= \sqrt{16\sin^2\left(\frac{2\pi}{5}\right)\sin^2\left(\frac{\pi}{5}\right) + 16\cos^2\left(\frac{2\pi}{5}\right)\sin^2\left(\frac{\pi}{5}\right)}$$

$$= 4\sin\left(\frac{\pi}{5}\right)\sqrt{\sin^2\left(\frac{2\pi}{5}\right) + \cos^2\left(\frac{2\pi}{5}\right)} = 4\sin\left(\frac{\pi}{5}\right) \quad \text{(shown)}$$



$$\theta = \tan^{-1}\left[\frac{4\cos\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right)}{4\sin\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right)}\right] = \tan^{-1}\left[\cot\left(\frac{2\pi}{5}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{2\pi}{5}\right)\right] = \frac{\pi}{10} \qquad \left[\because \tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha\right]$$

$$\therefore \ \arg(z_1 - z_2) = -\frac{\pi}{10} \text{ (shown)}$$